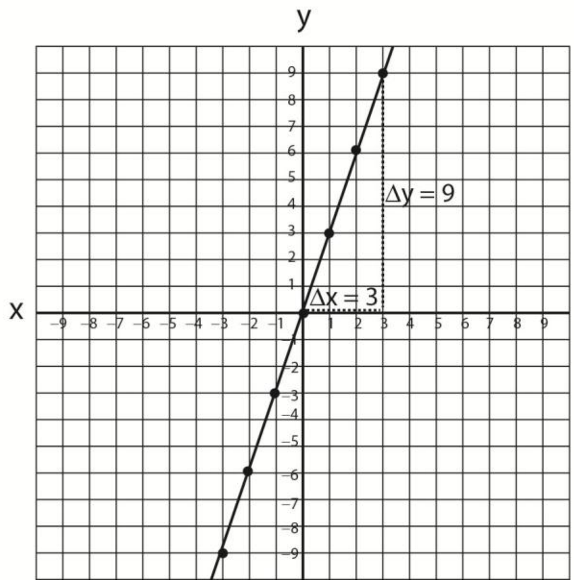
Functions, data tables, and graphing

1. Make a data table showing the values of x and y for the following functions, for each integer value of x from -3 to +3. (Hint: if necessary, start by algebraically manipulating the equation into a form that allows you to easily find the value of y). What are the slopes and y-intercepts of equations (a) and (b)? What type of function is equation (c)?
2. Graph the functions from Problem 1. You may use the same set of axes to plot all three functions. Label the x and y axes, and show the x and y scale that you have chosen. (You can use graph paper, or just draw the graph on plain paper that you have drawn axes and hatch marks to show the scale).
   a. y = 3x

$y = 3x$

x	y
-3	-9
-2	-6
-1	-3
0	0
1	3
2	6
3	9



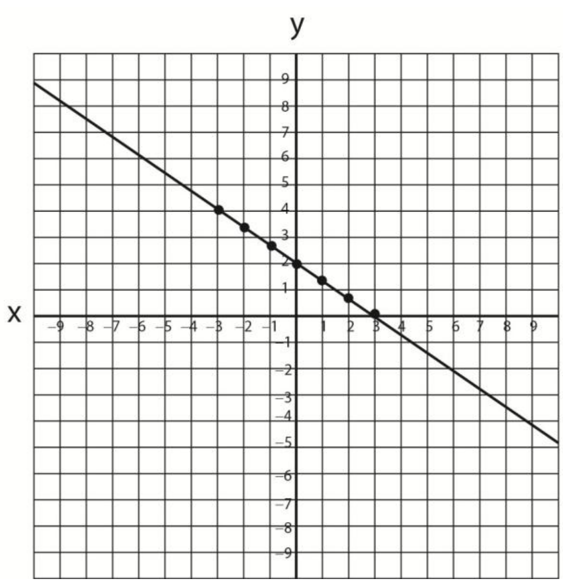
$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{9}{3} = 3$$

y = (slope)x + (y-intercept)
$y = 3x + 0$
slope = 3
y-intercept = 0

   b. 2x + 3y = 6

$2x + 3y = 6$
$3y = -2x + 6$
$y = -2/3x + 2$

x	y
-3	4
-2	3.33
-1	2.67
0	2
1	1.33
2	0.67
3	0



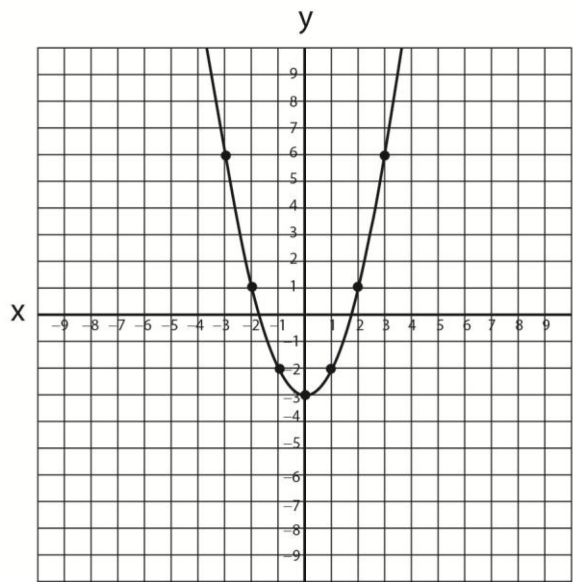
$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{-2}{3} = -2/3$$

y = (slope)x + (y-intercept)
$y = -2/3x + 2$
slope = –2/3
y-intercept = 2

c. $y + 3 = x^2$

$y + 3 = x^2$
$y = x^2 - 3$

x	y
-3	6
-2	1
-1	-2
0	-3
1	-2
2	1
3	6



Geometrical areas and volumes

3. Calculate the following:

a. The area of a square 5 feet on a side.

Area of square = (length of side)^2 = (5 ft)^2 = 25 ft^2

b. The circumference and area of a circle with a radius of 7.00 km.

Circumference circle of radius r = 2πr = 2π(7.00 km) = 14.0π = 44.0 km

Area circle of radius r = πr^2 = π(7.00 km)^2 = 49.0π = 154 km^2

c. The area of a triangle with a height of 30. m and a base of 5.0 m.

Area triangle = ½(base × height) = ½(30.0 m × 5.00 m) = 75 m^2

d. The surface area and volume of cube 3.0 inches on each edge.

Surface area cube = 6(length side)^2 = 6(3.0 in.)^2 = 54 in.^{2}

Volume cube = (length side)^3 = (3.0 in.)^3 = 27 in.^{3}

e. The radius and volume of a sphere with a *circumference* of 31.2 cm.

$$c = 2\pi r$$

$$r = \frac{c}{2\pi} = \frac{31.2}{2\pi} = 4.97 \text{ cm}$$

$$\text{volume sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4.97 \text{ cm})^3 = 514 \text{ cm}^3$$

Exponential and logarithm functions

4. Simplify the following expressions completely using the laws of exponents.
   a. $3^x \bullet 3^{-x} = 3^{x+(-x)} = 3^0 =$ **1**
   b. $(4^x)^y =$ $\mathbf{4^{x \bullet y}}$
   c. $3^3 \div 3^5 = 3^{3-5} = 3^{-2} = 1/(3^2) =$ **1/9**
   d. $e^a \bullet e^b \div e^c =$ $\mathbf{e^{a+b-c}}$
   e. $10^{bx} \bullet 10^{-cx} = 10^{bx+(-cx)} = 10^{bx-cx} =$ $\mathbf{10^{(b-c)x}}$
5. Simplify the following expressions completely using the properties of logarithms. (Note: "log x" written without a subscript is shorthand for $\log_{10}$ x, "ln" written without a subscript is short for $\log_e$ x.)
   a. log (m/n) = **log m – log n**
   b. log (m•n) = **log m + log n**
   c. log m^n = **n log m**
   d. ln e^{-n} = –n ln e = –n (1) = **–n**
   e. $\log_e$ (m•n) = $\log_e$ m + $\log_e$ n = **ln m + ln n**
6. Evaluate the following without using a calculator. (Hint: log 2 ≈ 0.30, log 3 ≈ 0.48, log 7 ≈ 0.85, ln 2 ≈ 0.69, ln 10 ≈ 2.3)
   a. $\log_3 27$ = power of 3 that yields 27 = $\log_3 3^?$ = $\log_3 3^3$ = 3 $\log_3$ 3 = 3 (1) = **3**
   b. $\log_8 2$ = $\log_8 8^?$ = $\log_8 8^{1/3}$ = 1/3 $\log_8$ 8 = 1/3 (1) = **1/3**
   c. log 6 = log 2•3 = log 2 + log 3 = 0.30 + 0.48 = **0.78**
   d. log 700 = log 7•100 = log 7 + log 100 = log 7 + log 10^2 = 0.85 + 2 = **2.85**
   e. log 3,000 = log 3•1000 = log 3 + log 1000 = log 7 + log 10^3 = 0.48 + 3 = **3.48**
   f. ln 8 = ln 2^3 = 3 ln 2 = 3•0.69 = **2.07 or 2.1**
   g. ln 0.1 = ln 10^{-1} = –1 ln 10 = –1(2.3) = **–2.3**

      or ln 0.1 = ln (1÷10) = ln 1 – ln 10 = 0 – 2.3 = **–2.3**

7. What is the doubling time (or halving time) if the exponential growth (or decay) rate is the following:
    a. 3.2% per year

        Population at time t = P(t)
        Population at time 0 = P_0
        r = rate
        t = time

        $P(t) = P_0\, e^{rt}$

        for doubling $P(t) = 2P_0$
        so $2P_0 = P_0\, e^{rt}$
        $2 = e^{rt}$
        $\ln 2 = \ln (e^{rt})$
        $\ln 2 = rt \ln e$
        $\ln 2 = rt$

        $$\frac{\ln 2}{r} = t$$

        $$\frac{0.69}{r} = t$$

        $$\mathbf{t = \frac{0.69}{r}}$$

        For 3.2% change per year or a rate of 0.032/yr

        $$t = \frac{0.69}{0.032/\text{yr}} = \mathbf{22\ years}$$

    b. 0.45/s

        $$t = \frac{0.69}{r} = \frac{0.69}{0.45/\text{s}} = \mathbf{1.5\ s}$$

    c. 0.0038/min

        $$t = \frac{0.69}{r} = \frac{0.69}{0.0038/\text{min}} = \mathbf{182\ min\ \ or\ \ 1.8 \times 10^2\ min}$$

8. A population of Canadian Geese grows exponentially at a rate of 10% per year.
    a. In about how many years will the population double?

        $$t = \frac{0.69}{r} = \frac{0.69}{0.10/\text{yr}} = \mathbf{6.9\ yr}$$

b. In about how many years will the population have doubled 20 times?

For 20 doublings $P(t) = 2^{20}P_0$

To see why, consider the initial population as 1. For 20 doublings,

1 to 2 to 4 to 8 to 16 to 32 to 64 to 128 to 256 to 512 to 1024 to 2048 to 4096 to 8192 to 16384 to 32768 to 65536 to 131072 to 262144 to 524288 to 1048576

$1048576 = 2^{20}$.

so $P(t) = 2^{20}P_0 = P_0\, e^{rt}$

$2^{20} = e^{rt}$

$\ln 2^{20} = \ln (e^{rt})$

$20 \ln 2 = rt$

$20(0.69) = 13.8 = rt$

$t = 13.8/r = 13.8/0.10/\text{yr} = 138\ \text{yr}$

c. About how many times the original population will the geese population be after 20 doublings? (Hint: Use the fact that $2^{10} = 1024 \approx 10^3$ to calculate the population).

2^{20} or 1048576 *See above*

Trigonometry

9. Convert the following angles from degrees to radians and vice versa:

a. 180°

$$? \text{ radians} = 180° \left(\frac{\pi}{180°} \right) = \pi$$

b. 270°

$$? \text{ radians} = 270° \left(\frac{\pi}{180°} \right) = 1.5\pi$$

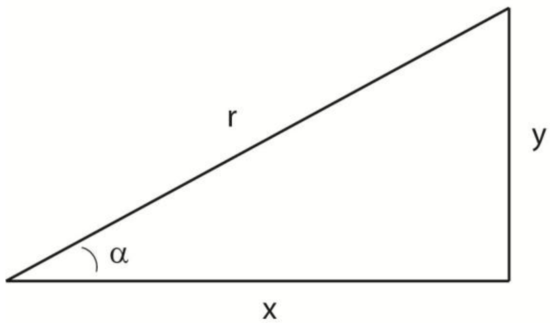
c. 4π/3

$$? \text{ degrees} = 4\pi/3 \left(\frac{180°}{\pi} \right) = 240°$$

d. −π

$$? \text{ degrees} = -\pi \left(\frac{180°}{\pi} \right) = -180°$$

10. For the right triangle below, write the formulas for sin α, cos α, and tan α in terms of the lengths of the sides x, y, and r.



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$$\sin \alpha = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r}$$

$$\cos \alpha = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r}$$

$$\tan \alpha = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x}$$

11. Write the formula for tan α in terms of sin α and cos α.

$$\tan \alpha = \frac{y}{x} = \frac{y/r}{x/r} = \frac{\sin \alpha}{\cos \alpha}$$

12. If angle α equals 60º and side r has a length of 5.00 miles, what are the lengths of sides x and y, respectively? [Hint: cos 30º = 0.500, sin 60º = 0.866).] Check your answer using the Pythagorean Theorem.

$$\sin \alpha = \frac{y}{r}$$

$$y = r \sin \alpha = 5.00 \text{ mi } (0.866) = 4.33 \text{ mi}$$

$$\cos \alpha = \frac{x}{r}$$

$$x = r \sin \alpha = 5.00 \text{ mi } (0.500) = 2.50 \text{ mi}$$

$$x^2 + y^2 = r^2$$
$$2.50^2 + 4.33^2 = 25.0 = 5^2$$