Earths Energy Balance Models

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Outline

- Stock Flow and temperature
- Earth as a black body
- Equation models for earth's temperature
  - Albedo effect
  - Greenhouse effect
- Balancing earth's energy flows
- Exam questions

EARTHS ENERGY STOCK AND FLOW

How does earth maintain an energy balance?

a stock & flow problem → $dS/dt = F_{in} - F_{out}$

Stock of what?

thermal energy

Inflows of what?

F_{in} *= solar flux, i.e. EM radiation in form of visible light*

Can we assume S is approximately at equilibrium?

Yes, because if not, earth would heat up or cool down

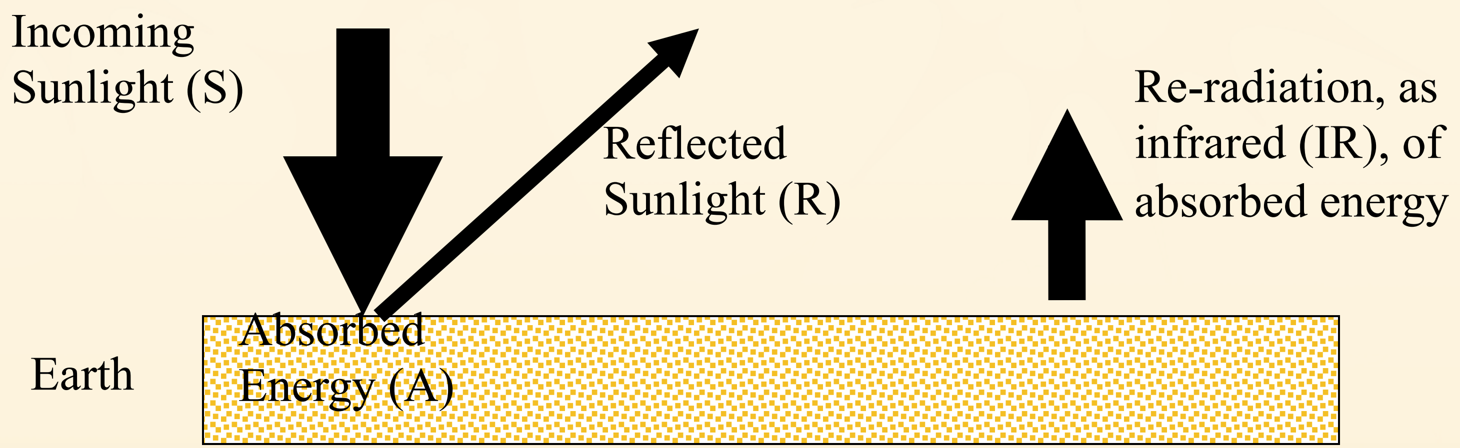
→ $dS/dt = 0$, $F_{in} = F_{out}$

Outflows of what?

(1) some solar energy is reflected back to space

(2) What happens to the rest? Can it be conducted away from earth? Can it be convected away? (No, space is a vacuum). So that leaves radiation: It is re-radiated as EM radiation in form of IR (heat)

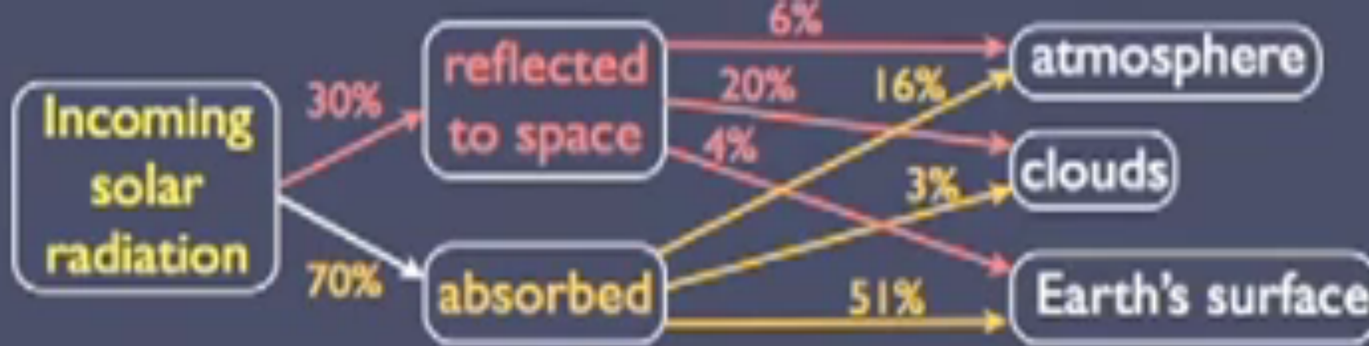
Simple box model: Energy flows at equilibrium

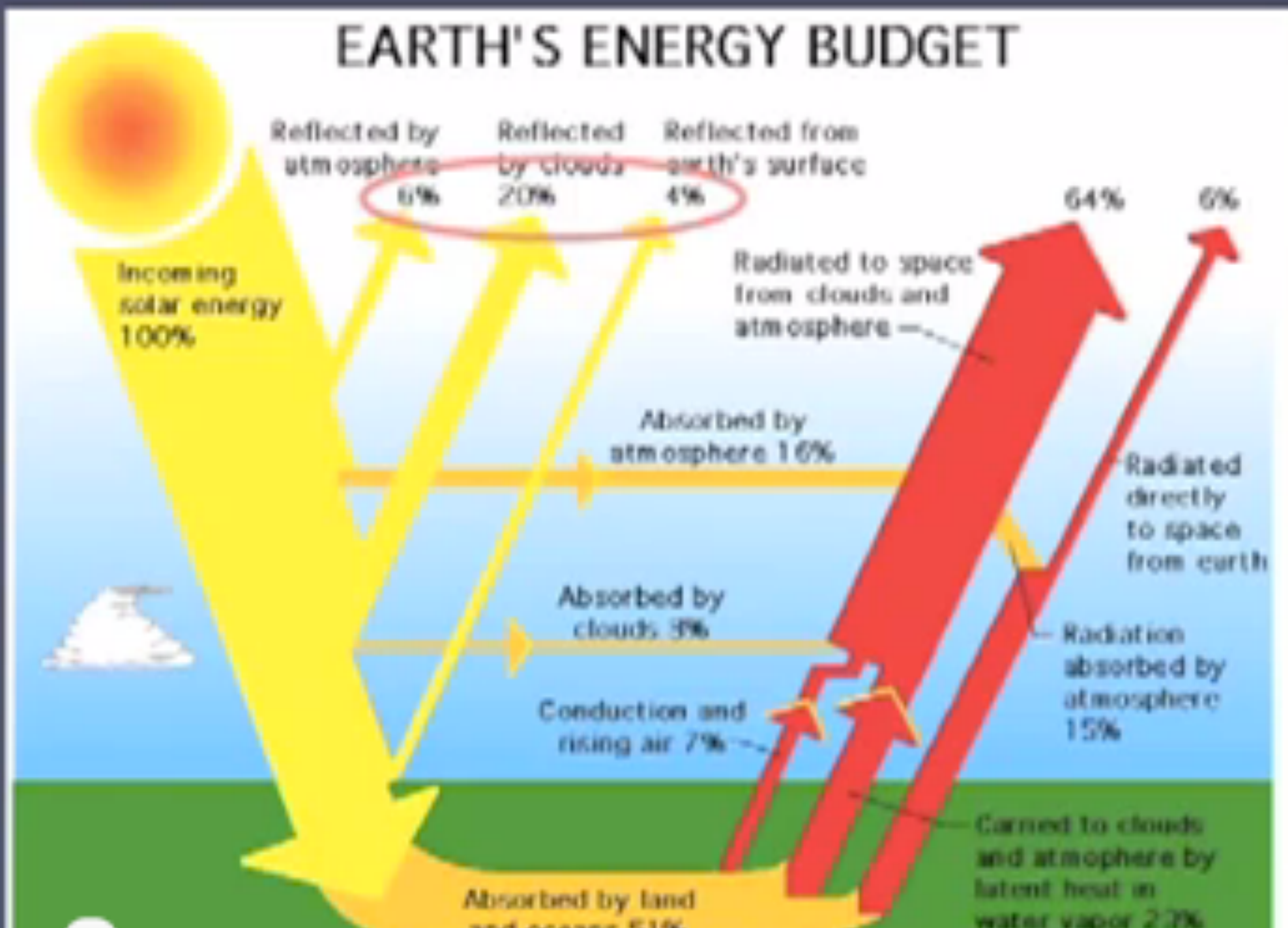


$A = S - R$ (conservation of energy)

$I = A$ (equilibrium assumption)

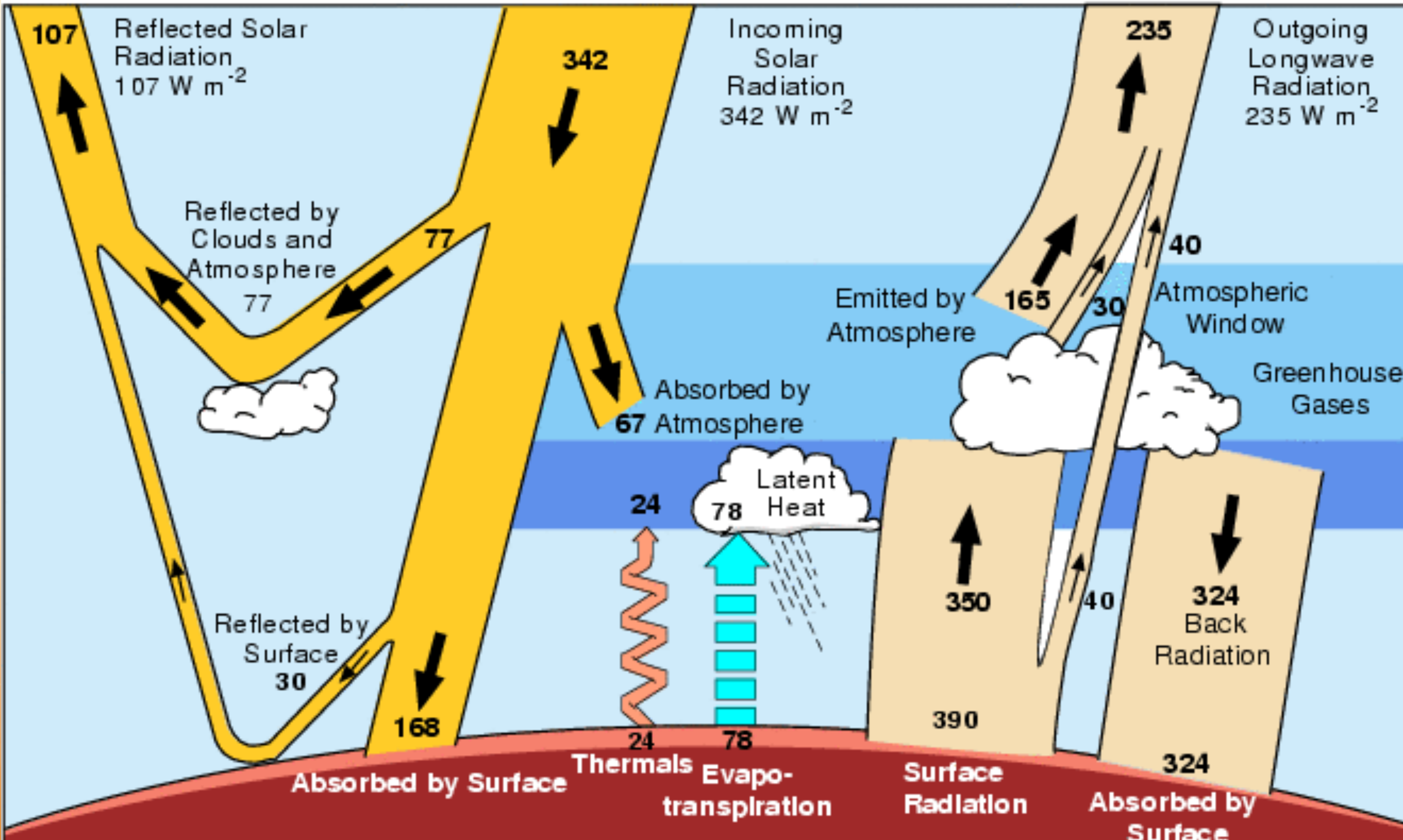
$I = S - R$ (combining the two)





% reflected
= albedo
= 30%

Global Heat Flows



Kiehl and Trenberth 1997

Energy flow to temperature

Steady state:

- The energy re-radiated by Earth to space is equal to the amount of incoming solar absorbed. If this were not true, the Earth would keep getting hotter and hotter.

How can we use this fact to find Earth's "temperature"?

- The blackbody radiation law, which relates the thermal energy radiated by a body to its temperature:

$$I = \sigma T^4$$

where "**I" is the infrared energy leaving Earth**, which in steady state = the solar energy absorbed by Earth (including atm) = 342 W/m^2 x (1 - 0.31) = **235 W/m^2**

T is absolute temperature in Kelvins = (°C + 273)

σ **(the Stefan-Boltzmann constant) = 5.67×10^{-8} W/m^2K^4**

Digression on Heat transfer

There are only 3 mechanisms of heat transfer:

1. **Conduction**
   - Thermal motion transferred from atom to atom in materials
   - Ex. Put your hand on a cold railing, it conducts heat from your hand, makes your hand feel cold
2. **Convection**
   - Thermal energy carried by the motion of fluids
   - Ex. Rising air currents that hawks coast on
3. **Radiation**
   - Transfer of energy via electromagnetic waves
   - Ex. The sun on your face
   - Energy of 1 photon ∝ frequency, $E = h\nu$
   - Total E = energy of photons x number of photons

Combine all three into Heat-Transfer Process:

$$q = \frac{A(T_i - T_o)}{R}$$

q= heat transfer rate through wall(Btu/hr)
A = wall area (m^2)
T_i = air temperature on one side of wall
T_o = ambient air temperature
R = overall thermal resistance (m^2 - °C/W)

Conduction, Convection and radiation

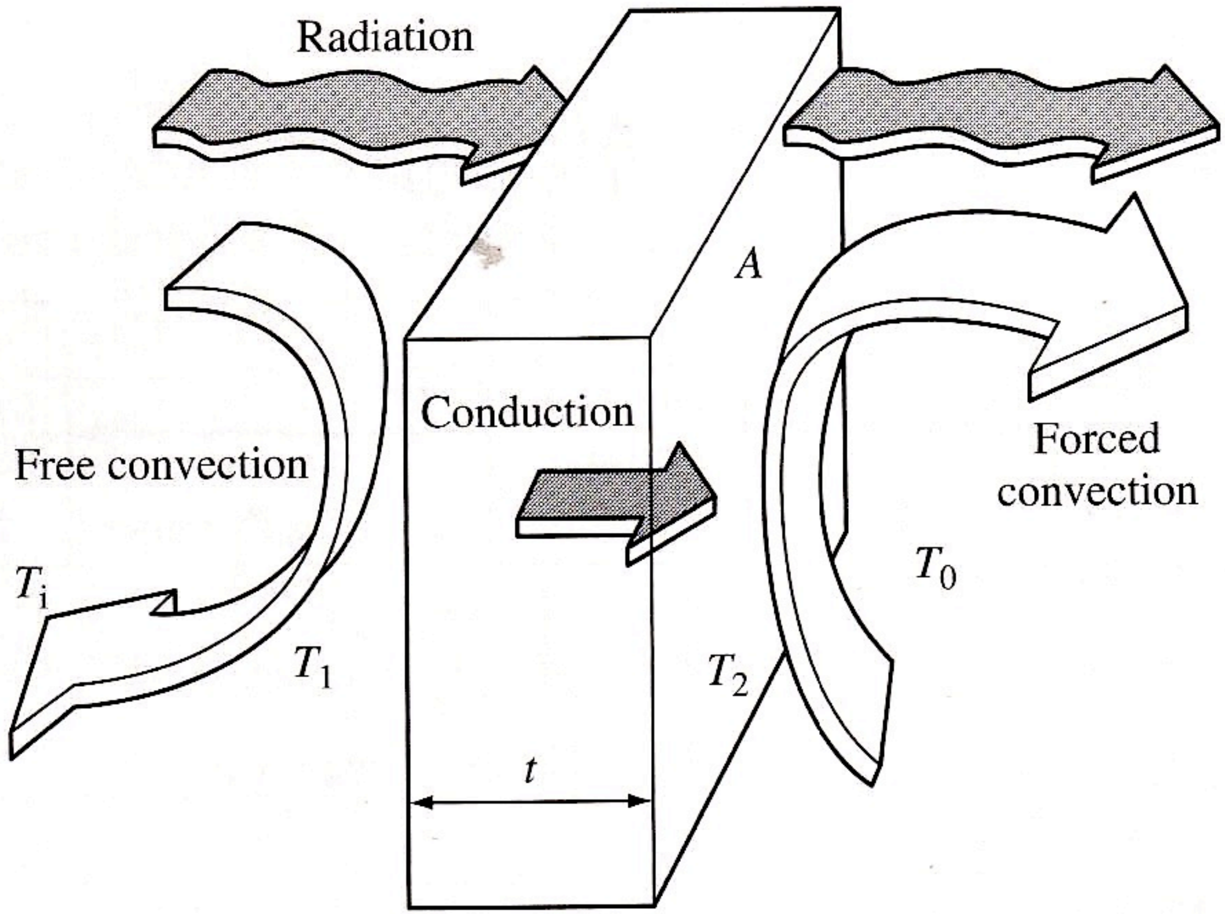


FIGURE 1.18 Heat transfer through a simple wall.

Ela

Consider a Blackbody Earth

- The energy radiated by an object, **E = blackbody emission rate (W)**, to its environment is proportional to its **temperature T**, its **surface area A**, and a property called **emissivity = ε**

$$E = \sigma \varepsilon A T^4$$

***Remember:** σ = stefan-boltzmann constant = 5.67×10^{-8} W/m^2-K^4

- **Emissivity** depends on factors such as temperature, emission, and wavelength
  - The emissivity in general depends on the thickness of material
  - Idealized black body has a emissivity equal to 1
- **Black body:** A black body is an idealized physical body that absorbs all incident electromagnetic radiation regardless of frequency or angle of incidence
  - 1) **ideal emitter**- emits as much or more energy at every frequency then any other body at the same temperature
  - 2) **diffuse emitter**-energy is radiated isotropically (independent of direction)

Consider a Blackbody Earth cont.

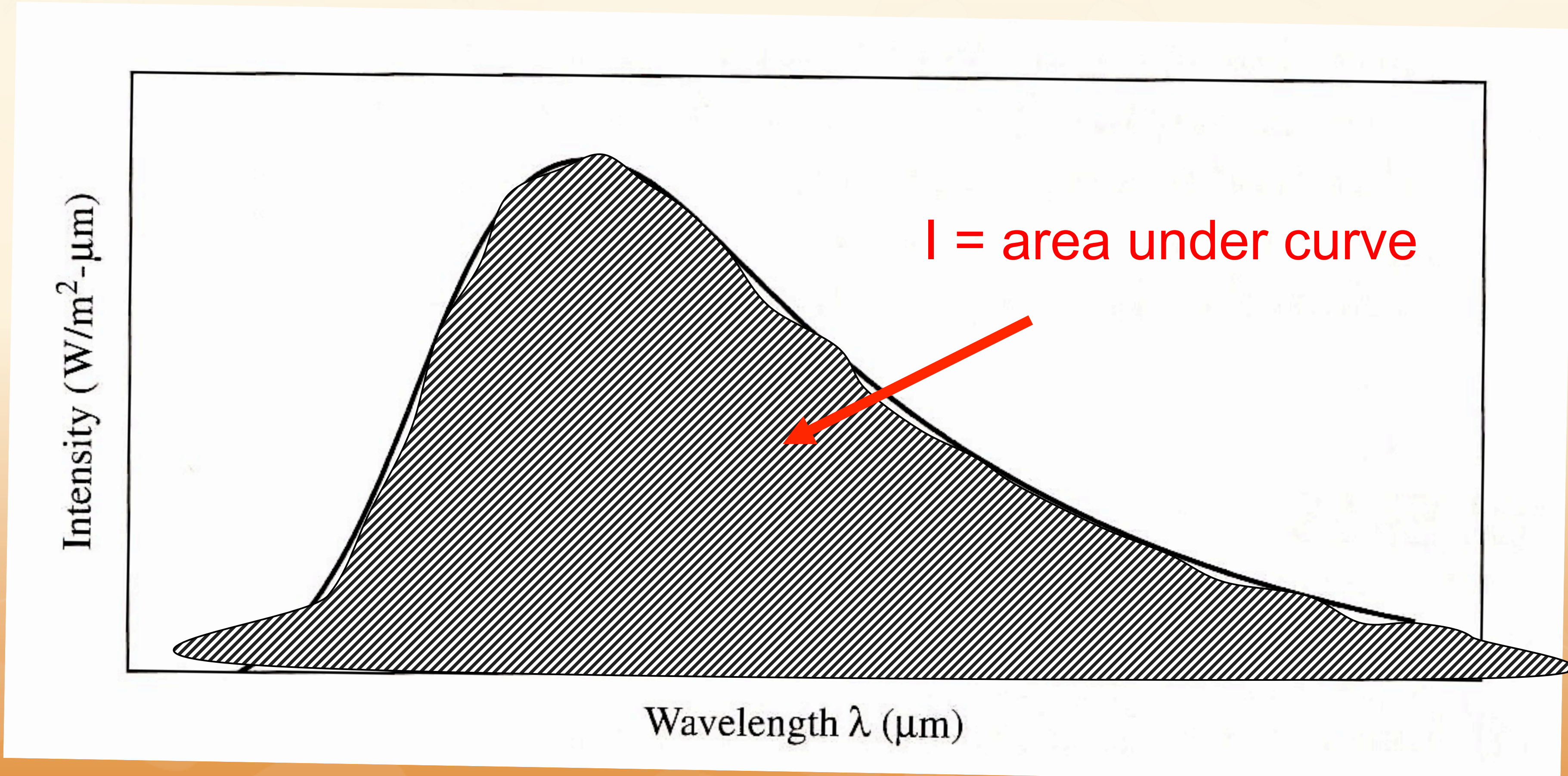
Blackbody radiation also holds in reverse - the amount of energy an object absorbs by radiation from its environment:

$$E = \sigma \, \varepsilon \, A \, T^4_{env}$$

T is in kelvins, so everything with a T > 0 radiates. There is a net flow from warm objects to cold objects, as the 2nd law of Thermo dictates. If we are warm and the window is cold because it's cold outside, we lose heat to the window. If we are at the same T, there is no net flow.

$$E_{net} = \sigma \, \varepsilon \, A \, (T^4_{env} - T^4)$$

Stefan-Boltzmann Law: $I = \frac{E}{A} = \sigma T^4$



Masters & Ela

Blackbody radiation cont.

Perfect Blackbody:

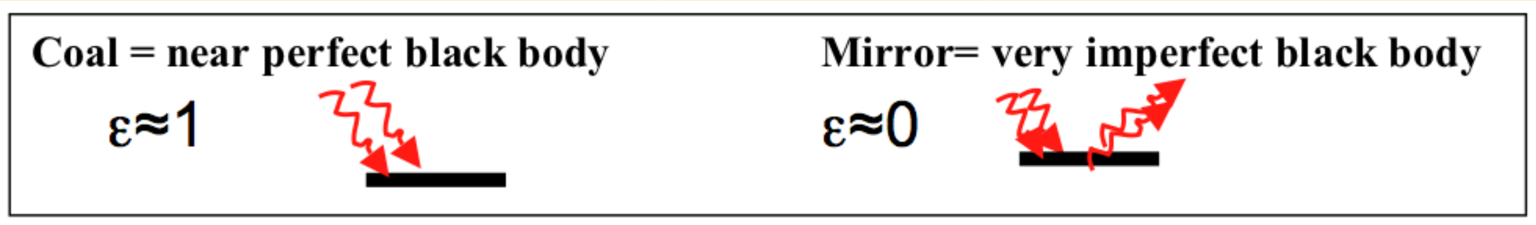
Total radiation = P = $\sigma\ \varepsilon\ A\ T^4$ (W)

Radiation flux (power per unit area) = P/A = $\sigma\ \varepsilon T^4$ (W/m^2)

$\varepsilon = \varepsilon(\lambda)$ i.e. ε varies according to the wavelength of the radiation:

A perfect "blackbody" at a given wavelength is one with $\varepsilon = 1$, meaning it radiates (or absorbs) the maximum possible amount at that wavelength. $\varepsilon = 0$ is the opposite, a bad radiator/absorber. All real objects are somewhere $0< \varepsilon<1$

Ex. at visible wavelengths, black cars get hotter in the sun than identical white cars.



Earth is a good radiator at IR wavelengths, so for now assume ε(IR) ≈ 1 (actually ε(IR) ≈ 0.95).

Earth's emissivity = $\varepsilon_{earth} \approx 1$

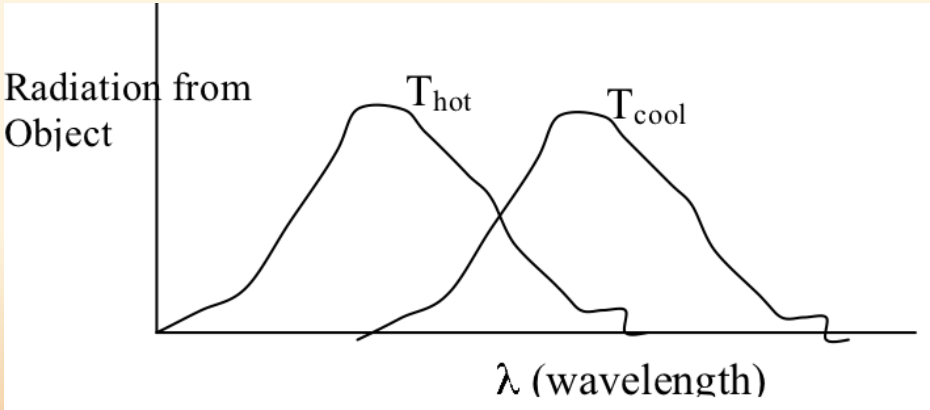
Earth's radiation flux = P/A ≈ σT^4

Blackbody radiation cont.

Black Body Curve

For every temperature, there is a wavelength distribution.

λ_{max} • T = constant Temperature goes up, wavelength goes down (Freq up)



Visible light hits earth and IR radiates. The sun is so hot, it emits at **λ = 200 to 800 nm** This is roughly the same as the visible range.

Earth thermal λ = 1000 to 20,000 nm. This is the IR re-radiated

Note:

***The earth is not actually a black body—(Latent heat, Convection, IR window..explained a litte more later)

Assuming the Earth is a blackbody….

The Earth's Energy Balance

Solar Flux $\Omega \approx 1360\ W/m^2$

avg. over surface $\rightarrow \Omega/4 \approx 340\ W/m^2$

Albedo (reflectivity) $a \approx 0.3$ (avg.)

Blackbody Radiation: $P(W/m^2) = \sigma T^4$

$\sigma = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4$

$(1-a)\,\Omega/4 = \sigma T^4$ (energy conservation)

$$T^4 = \frac{(1-a)\,\Omega}{4\sigma} = \frac{0.7 \times 340\ W/m^2}{5.67 \times 10^{-8}\ W/m^2 \cdot K^4}$$

$\longrightarrow T = 255\ K$

Simple earth's temperature balance

The emission temperature is defined as

$$T_e = \left[\frac{(1-\alpha_p)S}{4\sigma}\right]^{\frac{1}{4}}, \qquad (1)$$

where α_p is the planetary albedo, S the solar flux, and σ the Stefan-Boltzmann constant.

***Important equation☺

Temperature of a bare Earth
(slightly more precise than previous slide)

- Radiation balance: earth's outgoing IR must equal incoming solar

  $I = \Omega - R = \Omega(1-\alpha) = 342\ (0.69) = 342 - 107 = 235\ W/m^2$

R = Net radiation balance

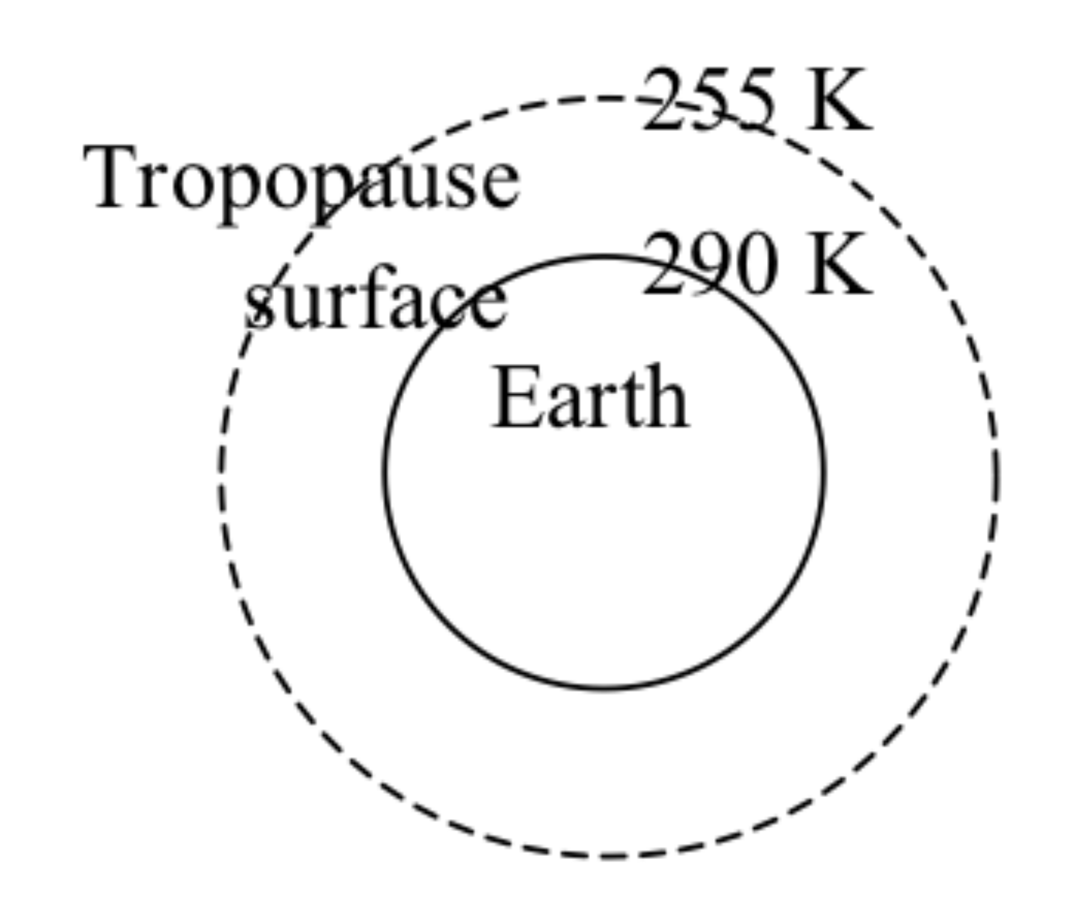
- Blackbody radiation law: $I = \sigma T^4$

  $T = (I/\sigma)^{1/4} = (235\ W/m2\ /\ 5.67\times10^{-8}\ W/m^2K^4)^{1/4}$

  $T = (41.4\times10^{8}\ K4)^{1/4} = 2.54\times10^{2} = 254\ K$

- T = **254 K** = **-19°C** (0°C = 273 K)
- A bare Earth (no atmosphere) with the reflectivity of the real Earth/atmosphere system would have an average equilibrium temperature of -19°C = 32°F – (9/5 x 19) = **-2°F**
- This is approximately the temperature of the **moon**.
- Since, in reality, the average T of the Earth is much higher than this (~15°C = 59°F), it's clear that the atmosphere makes a big difference.
- **Question: Where would you find this temperature?**

Earth's blackbody temperature

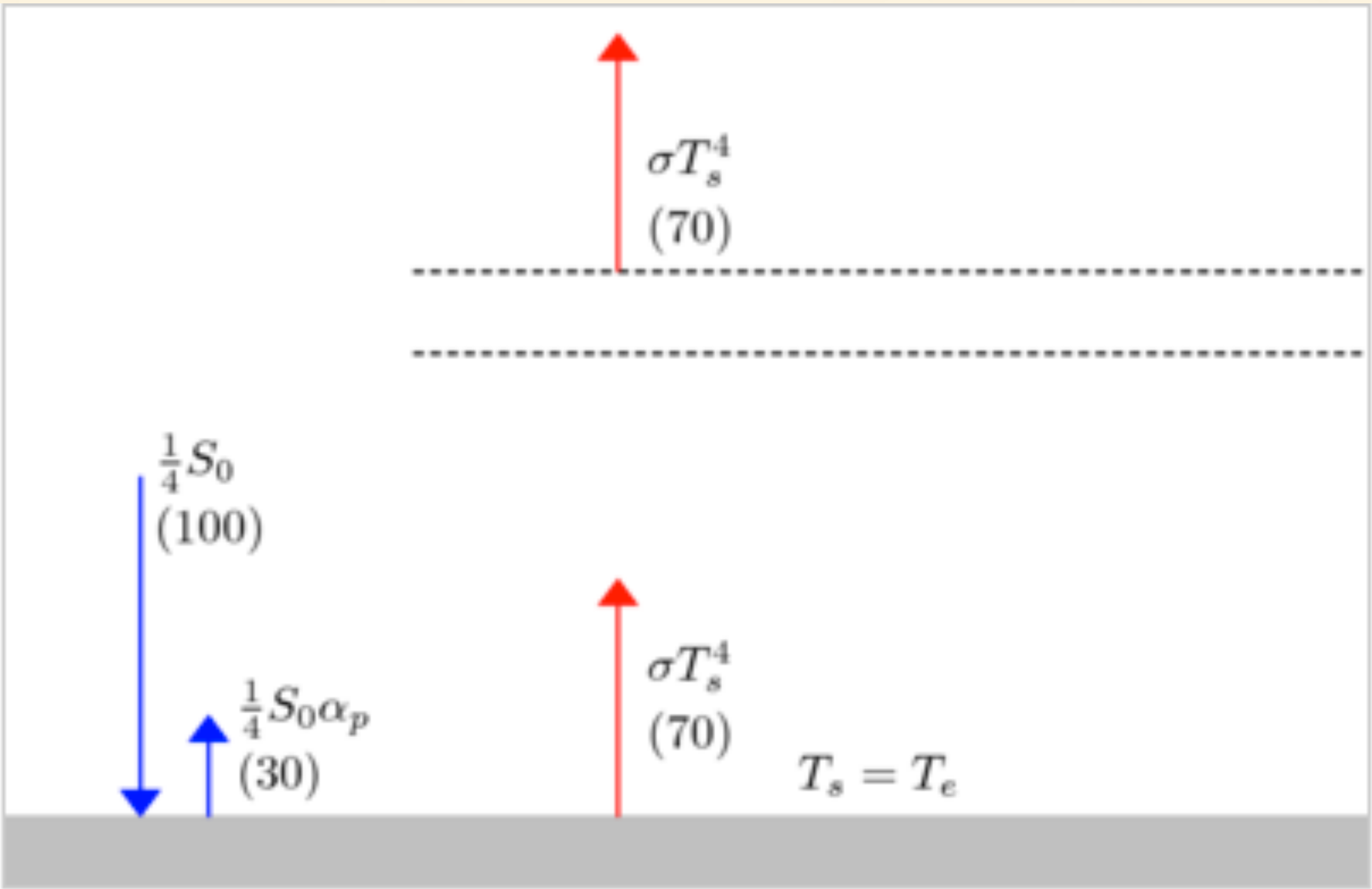


Tropopause: Atmosphere boundary between the troposphere and stratosphere
<u>Top of tropopause is 254K.</u> That's the T corresponding with the Wavelength of electromagnetic radiation emitted from earth.

Two-layer model of Greenhouse warming

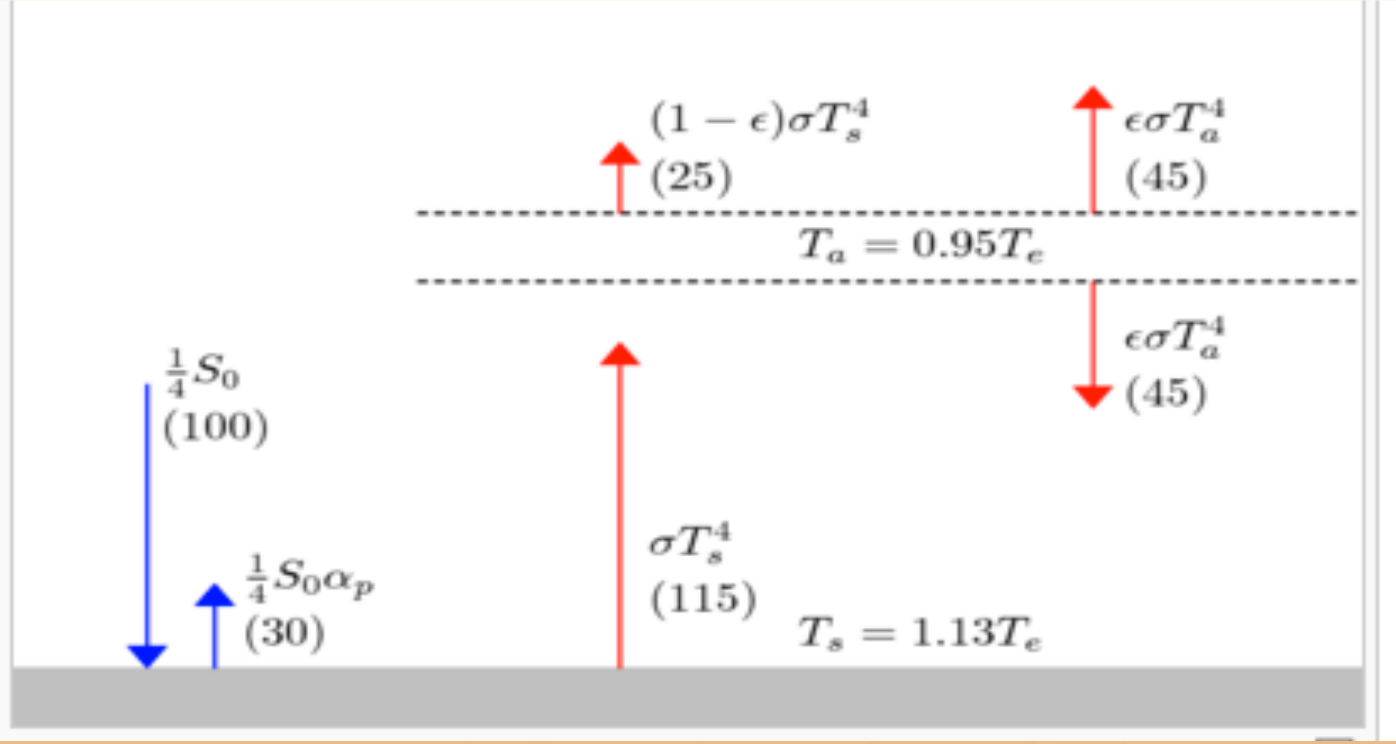
- A fraction α of atmospheric radiation is reflected back to space (α =0.31)
- Atmosphere transparent to remaining solar radiation
- Infrared radiation is absorbed in two layers centered at 0.5 and 3 km height acting as blackbodies

Earth's Energy Balance
No Greenhouse effect



The equilibrium solution with no greenhouse effect: ε=0

Earth's Energy Balance with greenhouse effect



Example of Earth having greenhouse gases creating an **emissivity of 0.78**

Digression on Latent Heat

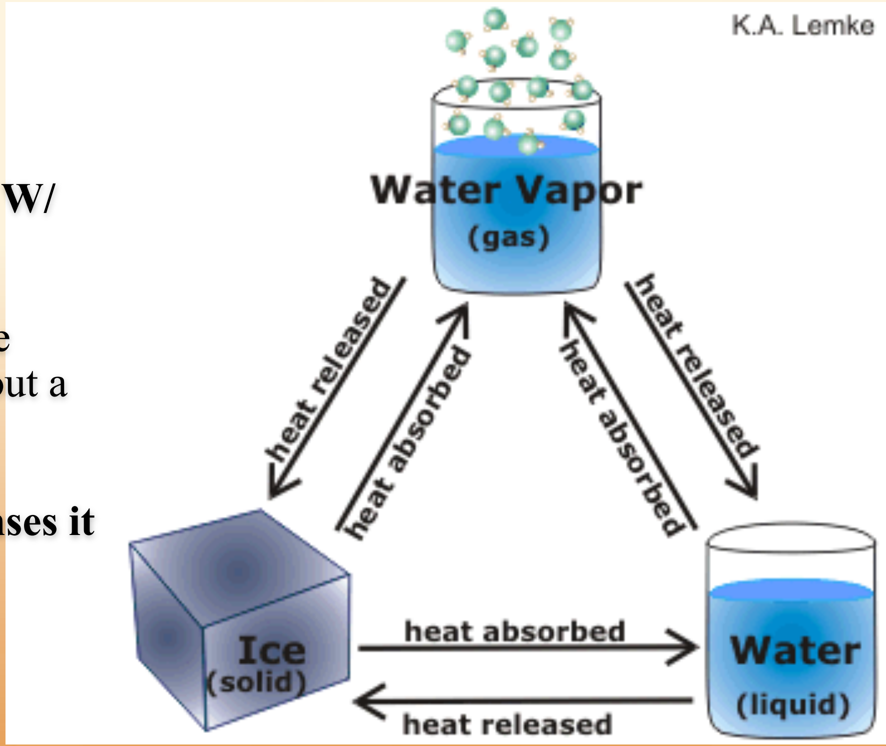
What happens if you add/subtract heat from water?
(1) Raise/lower temperature of water
specific heat: 1 kcal/kgºC = 4.2 kJ/kgºC @ 15ºC
(2) Cause a phase change
latent heat of vaporization/**condensation**:
2.26×10^6 J/kg @ 100°C (= 540 kcal/kg)
2.46×10^6 Jk/g @ 17°C
2.50×10^6 Jk/g @ 0°C

Of the 198 W/m^2 solar E that hits earth surface, 78 W/m^2 evaporates water.

This process of energy transformation cools the surface because the energy goes into the phase transition without a T change.

Vapor moves into atmosphere. When the air condenses it releases heat, so this process moves heat from the surface to the atmosphere.

This process drives the Hydrologic Cycle!



Model of T_s

T_s=Surface Temperature

Earth's surface can cool by:

1. Radiation (we already have that in the model)
2. Latent Heat Flow = 80 W/m^2 (ET)
3. Convective movement of air. Vertical Uplift of convective heat = 17 W/m^2
4. Sunlight absorbed in atmosphere before it hits surface (~ ½ in Lower layer, ~ ½ in upper layer)
5. 20 W/m^2 of IR from surface passes unabsorbed through the atmosphere. (5%)

- So, the actual emissivity to IR is $\varepsilon = 0.95$ (not $\varepsilon = 1$)

Model of T_S

$Ts = f(\Omega, \alpha, n, Fe, Fc, Fs, Fw)$
T = f (albedo, omega (solar flux), n (layers of the atmosphere), convection, latent heat, absorption, radiation absorbed in atm, leakage)

Calculate T_S with corrections:

Fc = convective (17W/m^2)
Fe = latent (80W/m^2)
Fs = sunlight absorbed in atmosphere (86W/m^2)
Fw = IR window (8-12 microns) (20W/m^2)

$$\sigma T_s^4 = 3\Omega/4\,(1-a) - [Fc + 1.5\,Fe + 1.7\,Fs + 2\,Fw] = 289K\ (16°C)$$

N-layer Radiation Balance Model

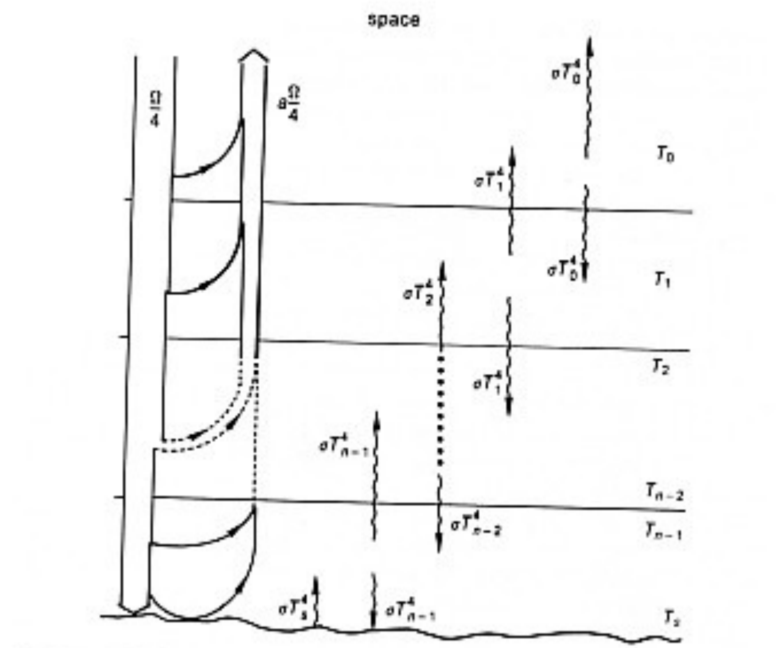


Figure III-5 The flow of energy in an n-layer atmosphere. The solar flux is Ω/4, a is the albedo, and the σT⁴ terms are infrared radiation fluxes (adapted from Goody and Walker, 1972).

- Energy balances for n=2
- Surface
- $\sigma T_s^4 = (\Omega/4)(1-\alpha)+\sigma T_1^4$
- Layer 1 (atm bot)
- $2\sigma T_1^4 = \sigma T_s^4 + \sigma T_0^4$
- Layer 0 (atm top)
- $2\sigma T_0^4 = \sigma T_1^4$
- Total atmosphere
- $(\Omega/4) = \alpha\,(\Omega/4) + \sigma T_0^4$

Substitution yields:

$T_s = (n+1)^{1/4}\, T_0$

For simple case with only albedo, Earth n=2

$T_s = (3)^{1/4}\, T_0$ = 336 K

$T_2 = (2)^{1/4}\, T_0$ = 303 K

$T_1 = (1)^{1/4}\, T_0$ = 255 K

Temperature with an atmosphere

- $I_{BB} = \sigma T^4 \Rightarrow T = 255$ K with no atmosphere.
- W/ atmosphere, more IR radiated - see previous slide
- $I_{SURFACE}$: 1.02 (E→A) + 0.12 (E→Space) ≈ 1.14 Ω/4
- $I_{SURFACE} = 1.14 * 342\ Wm^{-2} \approx 390\ Wm^{-2}$
- $T = \{[390Wm^{-2}]/[5.67*10^{-8}Wm^{-2}/K^4]\}^{1/4} = 288$ K
- T = 288K– 273K = 15 K above freezing, = 15 C

The difference between -19 C and +15 C is "the greenhouse effect", attributable to those constituents of the atmosphere that are transparent to incoming short-wave radiation (sunlight) but opaque to outgoing long-wave radiation (infrared), like glass in a greenhouse.

Simplified global temperature model

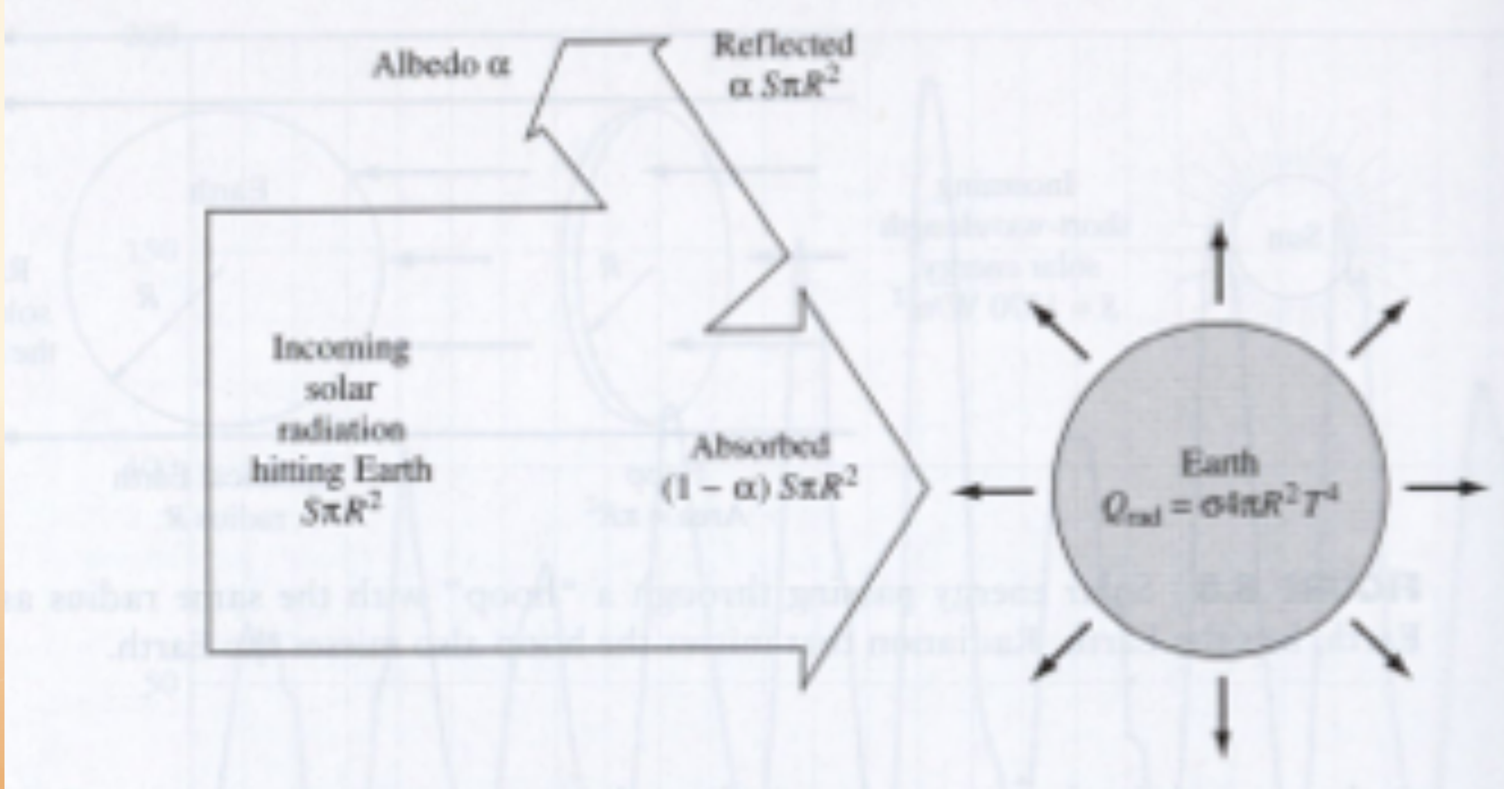


FIGURE 8.9 Simple global temperature model.

Earth's Albedo

Type of surface	Albedo (%)
Ocean	2 – 10
Forest	6 – 18
Cities	14 – 18
Grass	7 – 25
Soil	10 – 20
Grassland	16 – 20
Desert (sand)	35 – 45
Ice	20 – 70
Cloud (thin, thick stratus)	30, 60 – 70
Snow (old)	40 – 60
Snow (fresh)	75 – 95

Energy Flows at Earth's Surface

Energy inflows to Earth's surface:

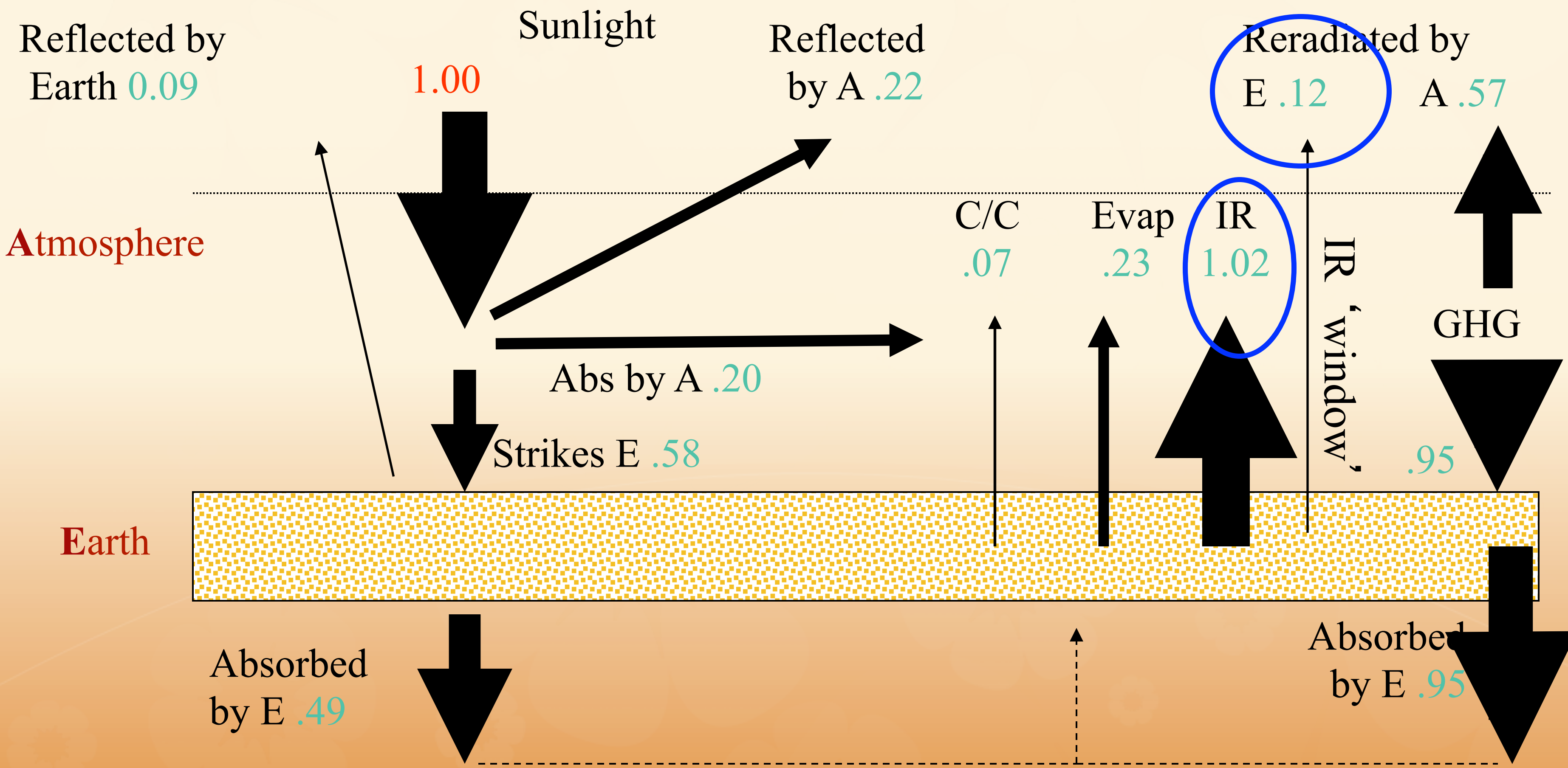
Solar radiation striking surface	198 W m^{-2}
Atmospheric IR back radiation	324 W m^{-2}
Tidal energy	0.005 W m^{-2}
Heat conduction from interior	0.06 W m^{-2}
Volcanoes and hot springs	0.0006 W m^{-2}
Anthropogenic activities	0.014 W m^{-2}

Energy outflows from Earth's surface:

Reflected solar radiation	30 W m^{-2}
Latent heat flux	78 W m^{-2}
Convection/conduction	24 W m^{-2}
IR radiation	390 W m^{-2}

Notice: $F_{in} = F_{out} = 522\ W/m^2$

Earth's energy balance, normalized. Incoming sunlight = 1.00 (= 342 W/m^2; see previous slide)



Energy moves by conduction & convection as well as by radiation.

Balancing the Earth's accounts:

In equilibrium, energy flows must balance for each component.

Location	Inflows (+)	Outflows (-)	Net
Surface	.58 + .95	.09 + .07 + .23 + 1.02 +.12	0
Atmosphere	.20 + .07 + . 23 + 1.02	.57 + .95	0
Earth/Atm System	1.00	.09 + .22 + .12 + .57	0

Exam questions

1. At present the emission temperature of the Earth is 255 K, and its albedo is 30%. How would the emission temperature change if:

   (a) the albedo were reduced to 10% (and all else were held fixed);

   (b) the infrared absorptivity of the atmosphere — ϵ in Fig.2.8 — were doubled, but albedo remains fixed at 30%.

2. Suppose that the Earth is, after all, flat. Specifically, consider it to be a thin circular disk (of radius 6370 km), orbiting the Sun at the same distance as the Earth; the planetary albedo is 30%. The vector normal to one face of this disk always points directly towards the Sun, and the disk is made of perfectly conducting material, so both faces of the disk are at the same temperature. Calculate the emission temperature of this disk, and compare with Eq.(2.4) for a spherical Earth.

Answer Question 1.

The emission temperature is defined as

$$T_e = \left[\frac{(1-\alpha_p)S}{4\sigma}\right]^{\frac{1}{4}}, \tag{1}$$

where α_p is the planetary albedo, S the solar flux, and σ the Stefan-Boltzmann constant.

(a) If albedo were reduced from $\alpha_p = 30\%$ to $\alpha'_p = 10\%$, the emission temperature would change from T_e (at present) to T'_e, where

$$\frac{T'_e}{T_e} = \left[\frac{1-\alpha'_p}{1-\alpha_p}\right]^{\frac{1}{4}} = \left[\frac{0.9}{0.7}\right]^{\frac{1}{4}} = 1.0648\ ,$$

so the new emission temperature would be $255 \times 1.0648 = 271.5$K.

(b) Emission temperature—the temperature at which the Earth emits to space—would not change at all if atmospheric IR opacity were doubled but albedo remained fixed. Emission temperature—unlike surface temperature—depends only on how much of the solar energy flux is absorbed by the Earth and, by (1), depends only on α_p, S, and σ.

Answer question 2: More practice

Incoming solar flux $S_0 = 1367\text{Wm}^{-2}$; planetary albedo $\alpha_p = 0.3$. Area of disk intercepting solar flux$= \pi a^2$. So,

$$\text{Net solar input} = S_0 \pi a^2 (1 - \alpha_p)\,.$$

Disk has temperature on both faces, so area emitting thermal radiation is $2\pi a^2$. Disk emits σT_e^4 per unit area, so

$$\text{Net thermal emission} \ = 2\pi a^2 \sigma T_e^4\,.$$

Balancing input and emission,

$$(1 - \alpha_p) S_0 \pi a^2 = 2\pi a^2 \sigma T_e^4\,,$$

i.e.,

$$T_e = \left[\frac{(1 - \alpha_p) S_0}{2\sigma}\right]^{\frac{1}{4}} = 303.1\text{K}\,.$$

The expression for T_e is a factor $2^{\frac{1}{4}}$ larger than we found for a spherical Earth—the disk has the same cross-section as the sphere (and so intercepts the same amount of solar radiation) but one-half of the surface area, so must increase T_e^4 by a factor of 2 to compensate.

Qualitative Exam Questions

- Explain the importance of the Albedo effect?
- Explain the difference in earth’s energy balance when assuming the earth to be a black body compared to its real emissivity?
- Explain the different ways earth’s surface cools?

References

- http://www.windows2universe.org/earth/climate/greenhouse_effect_gases.html
- Williams, Jim (2011). Earth's Energy Balance. http://en.wikipedia.org/wiki/Idealized_greenhouse_model

  http://en.wikipedia.org/wiki/Earth%27s_energy_budget