Calculations Relating to the Dimensions of the Physical World

Dimensions of the Physical World

- Classification of the physical world
  - **Geosphere** = the solid earth
  - **Hydrosphere** = the realms of water
  - **Atmosphere** = the gases surrounding the earth
  - **Biosphere** = the realm of life
- Three quantitative characteristics
  - **Stocks**: How much stuff is there?
  - **Flows**: What is rate of change (amount divided by time)?
  - **Characteristic times**: How long?
- **Goal**: to provide a basis for comparing and measuring human influences on the environment, e.g. to explore deforestation rates, we must first know how much forest there is.

A glimpse at some of the things we will be doing:

Calculate the approximate area of the earth covered by forest in square kilometers?

- Our calculations fall into two general categories
  - Equation-based (often with algebraic manipulation)
  - Unit conversions (using unit analysis)
- Possible general steps
  - Find information relating to the calculation.
    - Roughly 1/3 of Earth's land is forest land (including tropical, temperate, and boreal forests).
    - The Earth's surface is about 30% land and 70% water.
  - Clarify the problem…what do you need to know?
    - Need to know the surface area of Earth.

Problem: Calculate the approximate area of the earth covered by forest in square kilometers?

- Possible general steps (cont.)
  - Make assumptions and simplifications
    - Assume that Earth is a sphere.
  - Locate equations relating to problem

    Surface area of a sphere = A = $4\pi r^2 = \pi(2r)^2 = \pi d^2$

    Circumference of a sphere = C = $2\pi r = \pi d$
  - Find relevant data or make approximations
    - Because there are 24 time zones that are about 1000 miles each, the circumference of Earth is about 24,000 miles.

Problem: Calculate the area of the earth covered by forest in square kilometers?

- Possible general steps (cont.)
  - Manipulate equations to solve for your unknown, plug in values, do the calculation, and round your answer in such a way as to reflect the uncertainty of the values used in the calculation.

$$C = \pi d$$

$$d = \frac{C}{\pi} = \frac{24{,}000\text{ mi}}{\pi} = 7639.437\ldots\text{ mi} \approx 7.6 \times 10^3\text{ mi}$$

$$A = \pi d^2 = \pi(7.6 \times 10^3\text{ mi})^2 = 181458391.671\ldots\text{ mi}^2$$

$$\approx 1.8 \times 10^8\text{ mi}^2$$

Problem: Calculate the area of the earth covered by forest in square kilometers?

- Possible general steps (cont.)
  - Make necessary conversions

$$? \text{ km}^2 = 1.8 \times 10^8 \text{ mi}^2 \left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right)^2 = 465998580 \approx 4.7 \times 10^8 \text{ km}^2 \text{ Earth}$$

$$? \text{ km}^2 \text{ land} = 4.7 \times 10^8 \text{ km}^2 \text{ Earth} \left(\frac{30 \text{ km}^2 \text{ land}}{100 \text{ km}^2 \text{ Earth}}\right) \approx 1.4 \times 10^8 \text{ km}^2 \text{ land}$$

$$? \text{ km}^2 \text{ forest} = 1.4 \times 10^8 \text{ km}^2 \text{ land} \left(\frac{1 \text{ km}^2 \text{ forest}}{3 \text{ km}^2 \text{ land}}\right) \approx 4.7 \times 10^7 \text{ km}^2 \text{ forest}$$

or all in one

$$? \text{ km}^2 \text{ forest} = 1.8 \times 10^8 \text{ mi}^2 \text{ Earth} \left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right)^2 \left(\frac{30 \text{ km}^2 \text{ land}}{100 \text{ km}^2 \text{ Earth}}\right) \left(\frac{1 \text{ km}^2 \text{ forest}}{3 \text{ km}^2 \text{ land}}\right) \approx 4.7 \times 10^7 \text{ km}^2 \text{ forest}$$

Distribution of land by type of cover:
another simple set of round numbers
Forest / Other Productive Land / Wasteland ~ 1/3 each

- Forest 50×10^6 km^2
  - Tropical 25×10^6 km^2
  - Temperate and boreal 25×10^6 km^2
- Other productive land 50×10^6 km^2
  - Woodland, swamp, marsh 10×10^6 km^2
  - Cultivated land (2/3 cropped each year) 15×10^6 km^2
  - Grassland, savannah (much grazed) 25×10^6 km^2
- Wasteland 50×10^6 km^2
  - Ice 15×10^6 km^2
  - Tundra, permafrost, rock, lakes 15×10^6 km^2
  - Desert and near desert 20×10^6 km^2

Background: What do we need to know?

- Where to get information and equations
  - Appendix of Consider and Spherical Cow (COW)
  - Google
- How to do simple algebra
  - Whatever you do to one side of the equation, do the same to the other…many examples to follow.
- Common units
  - An Introduction to Chemistry (AIC) Section 1.4 and Appendix A
  - Environmental Engineering and Science (EES) pages 1-4
- How to make unit conversions
  - Unit analysis (Dimensional Analysis)

Background: What do we need to know?

- Types of unit conversions (AIC Chapter 8)
  - SI→SI (metric-metric)
  - English↔SI
  - Squared and cubed units
  - Density as a conversion factor
  - Percentages as conversion factors
  - Anything that can be expressed as “something per something” can be expressed as a ratio that can be used as a conversion factor.
- How to round off numbers to reflect the uncertainty of the values used in calculations
  - AIC Section 8.2
- Converting between decimal numbers and numbers expressed in scientific notation
  - AIC Appendix B

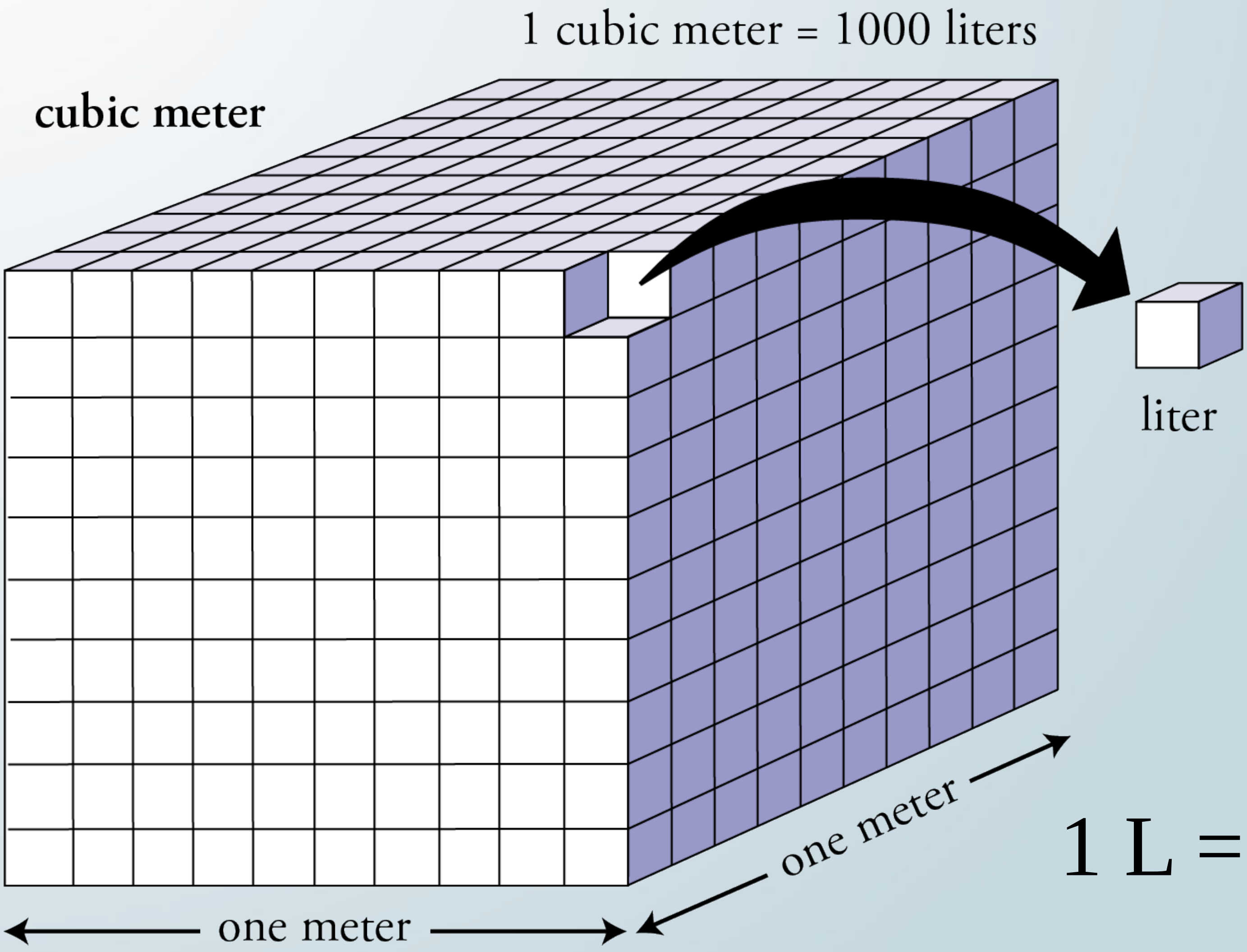
Values from Measurements

- A ***value*** is a quantitative description that includes both a unit and a number.
- For *100 meters*, the *meter* is a unit by which distance is measured, and the *100* is the number of units contained in the measured distance.
- ***Units*** are quantities defined by standards that people agree to use to compare one event or object to another.

Base Units for the International System of Measurement

- **Length meter, m**, the distance that light travels in a vacuum in 1/299,792,458 of a second
- **mass kilogram, kg**, the mass of a platinum-iridium alloy cylinder in a vault in France
- **time second, s**, the duration of 9,192,631,770 periods of the radiation emitted in a specified transition between energy levels of cesium-133
- **temperature kelvin, K**, 1/273.16 of the temperature difference between absolute zero and the triple point temperature of water
- **Amount of substance mole, mol**, the amount of substance that contains the same number of chemical units as there are atoms in 12 g of carbon-12.
- **Electric current ampere, A**, the amount of electric charge passing a point in an electric circuit per unit time with 6.241×10^{18} electrons, or one coulomb per second constituting one ampere.
- **Luminous intensity candela, Cd**, the luminous intensity of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity of $\frac{1}{683}$ watt per steradian.

Derived Unit



$$1\ \text{L} = 10^{-3}\ \text{m}^3$$

$$10^3\ \text{L} = 1\ \text{m}^3$$

Derived Units

$$\text{velocity} = \frac{\text{distance}}{\text{time}} = \frac{\text{m}}{\text{s}}$$

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}} = \frac{\text{m/s}}{\text{s}} = \frac{\text{m}}{\text{s}^2}$$

$$\text{force} = \text{mass} \bullet \text{acceleration} = \frac{\text{kg}\bullet\text{m}}{\text{s}^2} = \text{Newton, N}$$

$$\text{pressure} = \frac{\text{force}}{\text{area}} = \frac{\text{kg}\bullet\text{m}}{\text{s}^2} \bullet \frac{1}{\text{m}^2} = \frac{\text{kg}}{\text{m}\bullet\text{s}^2} = \text{Pascal, Pa}$$

$$\text{energy} = \text{force} \bullet \text{distance} = \frac{\text{kg}\bullet\text{m}}{\text{s}^2}\bullet\text{m} = \frac{\text{kg}\bullet\text{m}^2}{\text{s}^2} = \text{Joule, J}$$

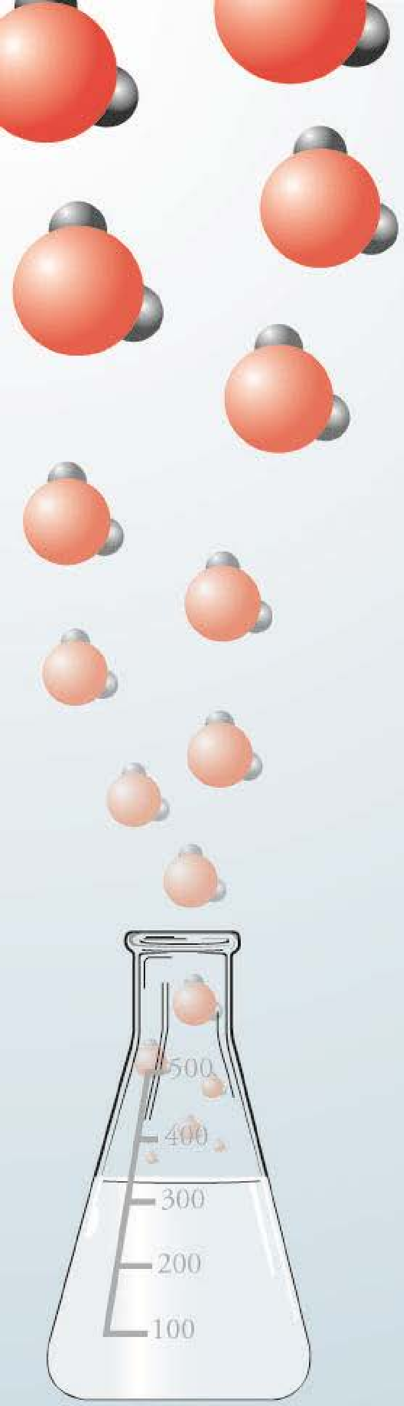
Most Important Derived Units

Force	newton, N = kg·m/s^2
Energy or Work	joule, J = N·m
Power	watt, W = J/s
Gas pressure	pascal, Pa = N/m^2

Normal room pressure is about 101,000 Pa

Radiation exposure	sievert, Sv = J/kg

3 Sv exposure leads to 50% chance of death

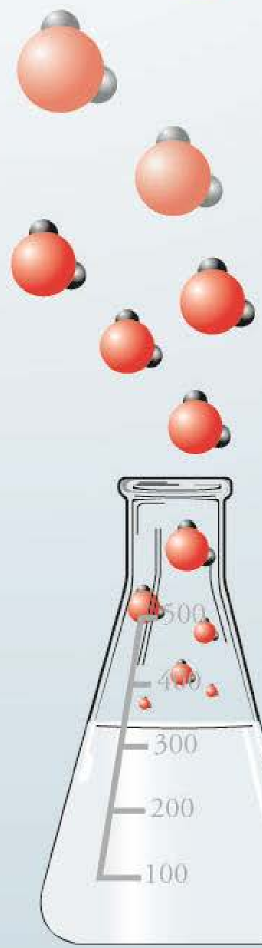


Metric Prefixes

Prefix	Abbreviation	Number
exa	E	10^{18}
peta	P	10^{15}
tera	**T**	**10^{12}**
giga	**G**	**10^{9} or 1,000,000,000**
mega	**M**	**10^{6} or 1,000,000**
kilo	**k**	**10^{3} or 1000**
centi	**c**	**10^{-2} or 0.01**
milli	**m**	**10^{-3} or 0.001**
micro	μ	10^{-6} or 0.000001
nano	n	10^{-9} or 0.000000001
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

Scientific (Exponential) Notation

- Numbers expressed in scientific notation have the following form.



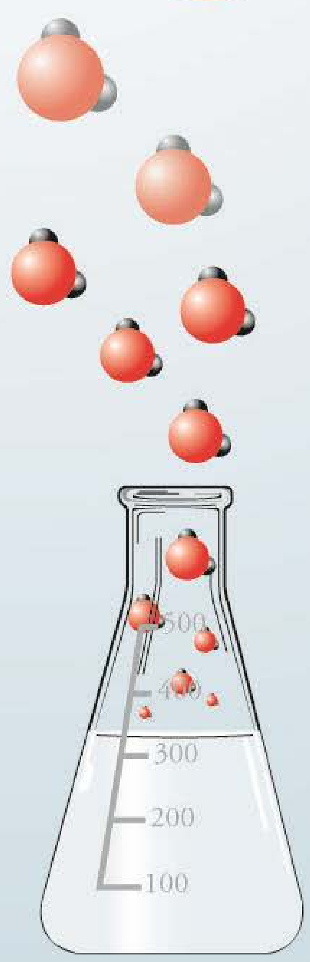
$$a \times 10^{b}$$

Exponent, a positive or negative integer

Exponential term

Coefficient,
a number with one nonzero digit
to the left of the decimal point

Scientific Notation (Example)



- 5.5×10^{21} carbon atoms in a 0.55 carat diamond.
  - 5.5 is the coefficient
  - 10^{21} is the exponential term
  - The ^{21} is the exponent.
- The coefficient usually has one nonzero digit to the left of the decimal point.

Uncertainty

- The coefficient reflects the number’s uncertainty.
- It is common to assume that coefficient is plus or minus one in the last position reported unless otherwise stated.
- Using this guideline, 5.5×10^{21} carbon atoms in a 0.55 carat diamond suggests that there are from 5.4×10^{21} to 5.6×10^{21} carbon atoms in the stone.

Size (Magnitude) of Number

- The exponential term shows the size or magnitude of the number.
- Positive exponents are used for large numbers. For example, the moon orbits the sun at 2.2×10^4 or 22,000 mi/hr.

$$2.2 \times 10^4 = 2.2 \times 10 \times 10 \times 10 \times 10 = 22{,}000$$

Negative exponents are used for small numbers. For example, A red blood cell has a diameter of about 5.6×10^{-4} or 0.00056 inches.

$$5.6 \times 10^{-4} = 5.6 \times \frac{1}{10^4} = \frac{5.6}{10 \times 10 \times 10 \times 10} = 0.00056$$

From Decimal Number to Scientific Notation

- Shift the decimal point until there is one nonzero number to the left of the decimal point, counting the number of positions the decimal point moves.
- Write the resulting coefficient times an exponential term in which the exponent is positive if the decimal point was moved to the left and negative if the decimal position was moved to the right. The number in the exponent is equal to the number of positions the decimal point was shifted.

From Decimal Number to Scientific Notation (Examples)

- For example, when 22,000 is converted to scientific notation, the decimal point is shifted four positions to the left so the exponential term has an exponent of 4.

$$22{,}000 = 2.2 \times 10^4$$

- When 0.00056 is converted to scientific notation, the decimal point is shifted four positions to the right so the exponential term has an exponent of -4.

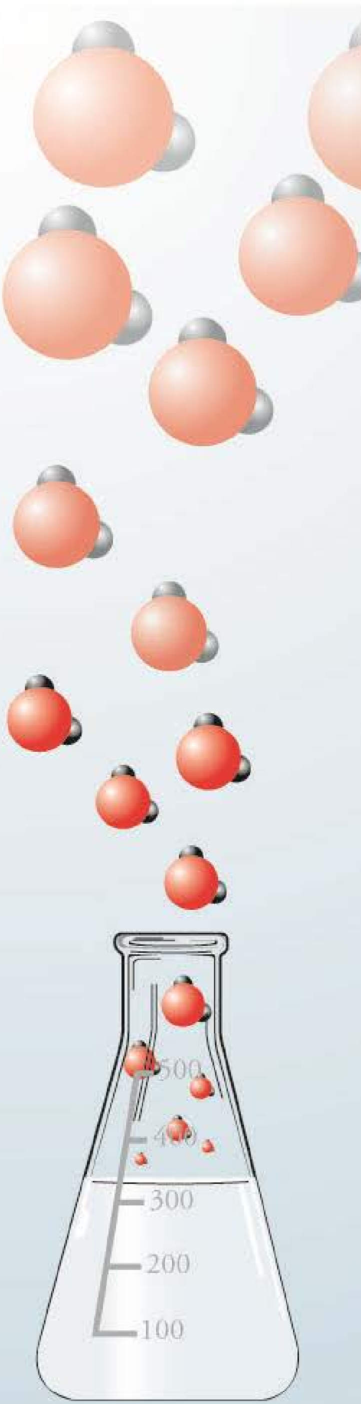
$$0.00056 = 5.6 \times 10^{-4}$$

Scientific Notation to Decimal Number

- Shift the decimal point in the coefficient to the right if the exponent is positive and to the left if it is negative.
- The number in the exponent tells you the number of positions to shift the decimal point.

2.2×10^{4} goes to 22,000

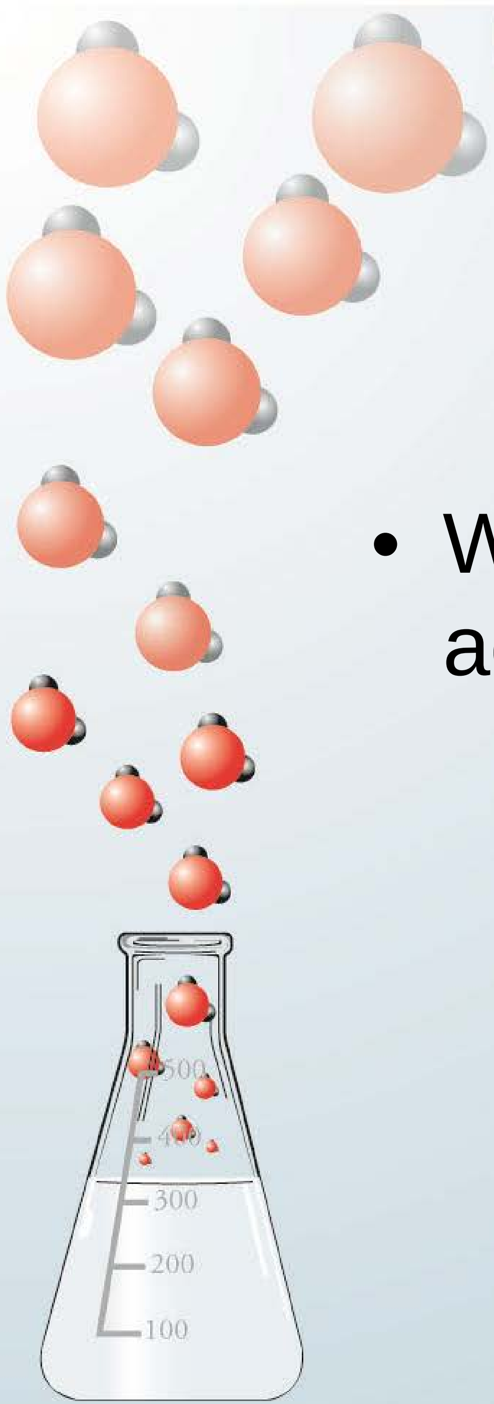
5.6×10^{-4} goes to 0.00056



Reasons for Using Scientific Notation

- **Convenience -** It takes a lot less time and space to report the mass of an electron as 9.1096×10^{-28}, rather than 0.00000000000000000000000000091096 g.
- **To more clearly report the uncertainty of a value -** The value 1.4×10^{3} kJ per peanut butter sandwich suggests that the energy from a typical peanut butter sandwich could range from 1.3×10^{3} kJ to 1.5×10^{3} kJ. If the value is reported as 1400 kJ, its uncertainty would not be so clear. It could be 1400 ± 1, 1400 ± 10, or 1400 ± 100.

Multiplying Exponential Terms



- When multiplying exponential terms, add exponents.

$$10^3 \times 10^6 = 10^{3+6} = 10^9$$

$$10^3 \times 10^{-6} = 10^{3+(-6)} = 10^{-3}$$

$$3.2 \times 10^{-4} \times 1.5 \times 10^9$$
$$= 3.2 \times 1.5 \times 10^{-4+9}$$
$$= 4.8 \times 10^5$$

When dividing exponential terms, subtract exponents.

$$\frac{10^{12}}{10^{3}} = 10^{12-3} = 10^{9}$$

$$\frac{10^{6}}{10^{-3}} = 10^{6-(-3)} = 10^{9}$$

$$\frac{9.0 \times 10^{11}}{1.5 \times 10^{-6}} = \frac{9.0}{1.5} \times 10^{11-(-6)} = 6.0 \times 10^{17}$$

$$\frac{10^{2} \cdot 10^{-3}}{10^{6}} = 10^{2+(-3)-6} = 10^{-7}$$

$$\frac{1.5 \times 10^{4} \cdot 4.0 \times 10^{5}}{2.0 \times 10^{12} \cdot 10^{3}} = \frac{1.5 \cdot 4.0}{2.0} \times 10^{4+5-12-3} = 3.0 \times 10^{-6}$$

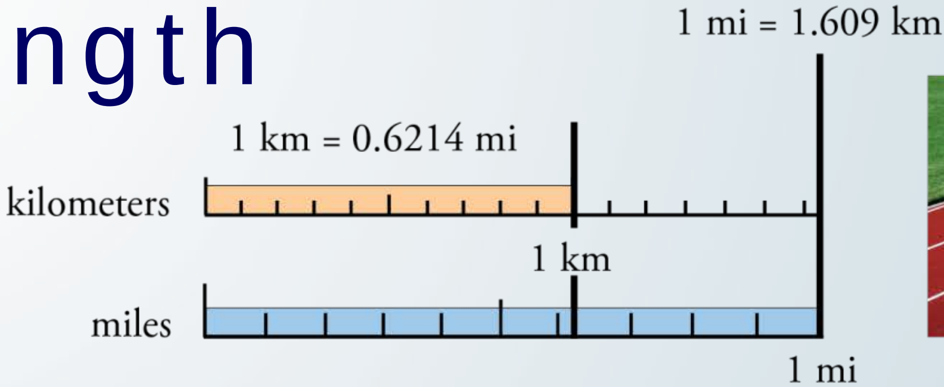
Raising Exponential Terms to a Power

- When raising exponential terms to a power, multiply exponents.

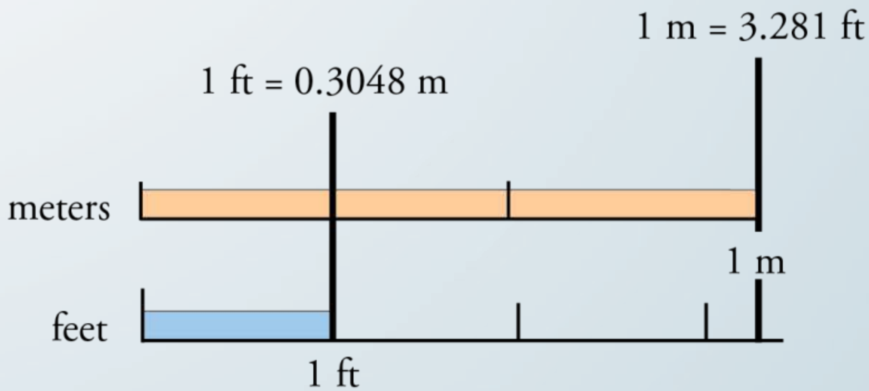
$$(10^4)^3 = 10^{4 \cdot 3} = 10^{12}$$

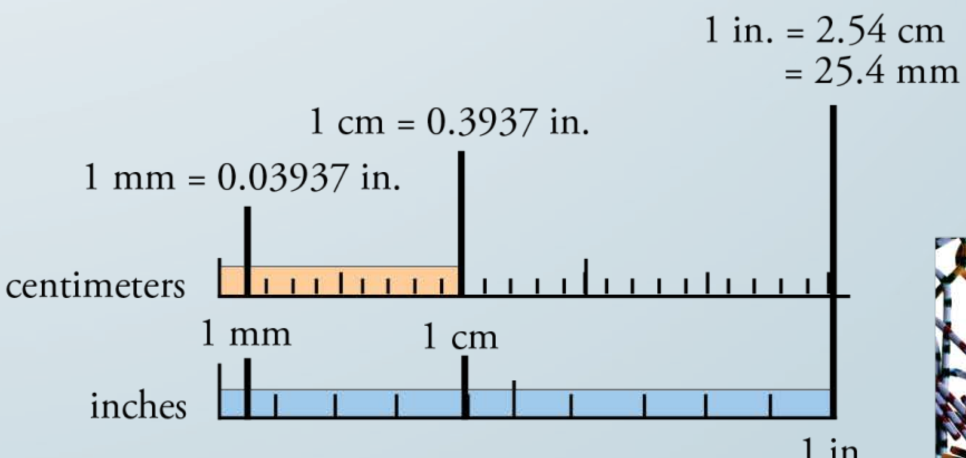
$$(3 \times 10^5)^2 = (3)^2 \times (10^5)^2 = 9 \times 10^{10}$$

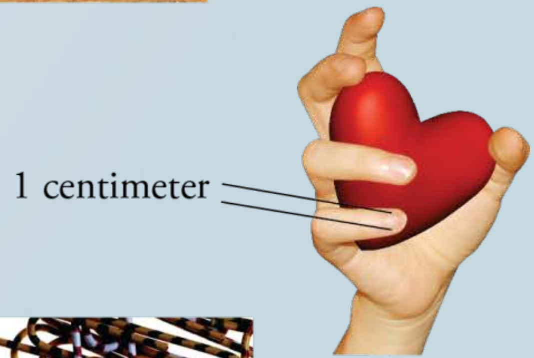
Length



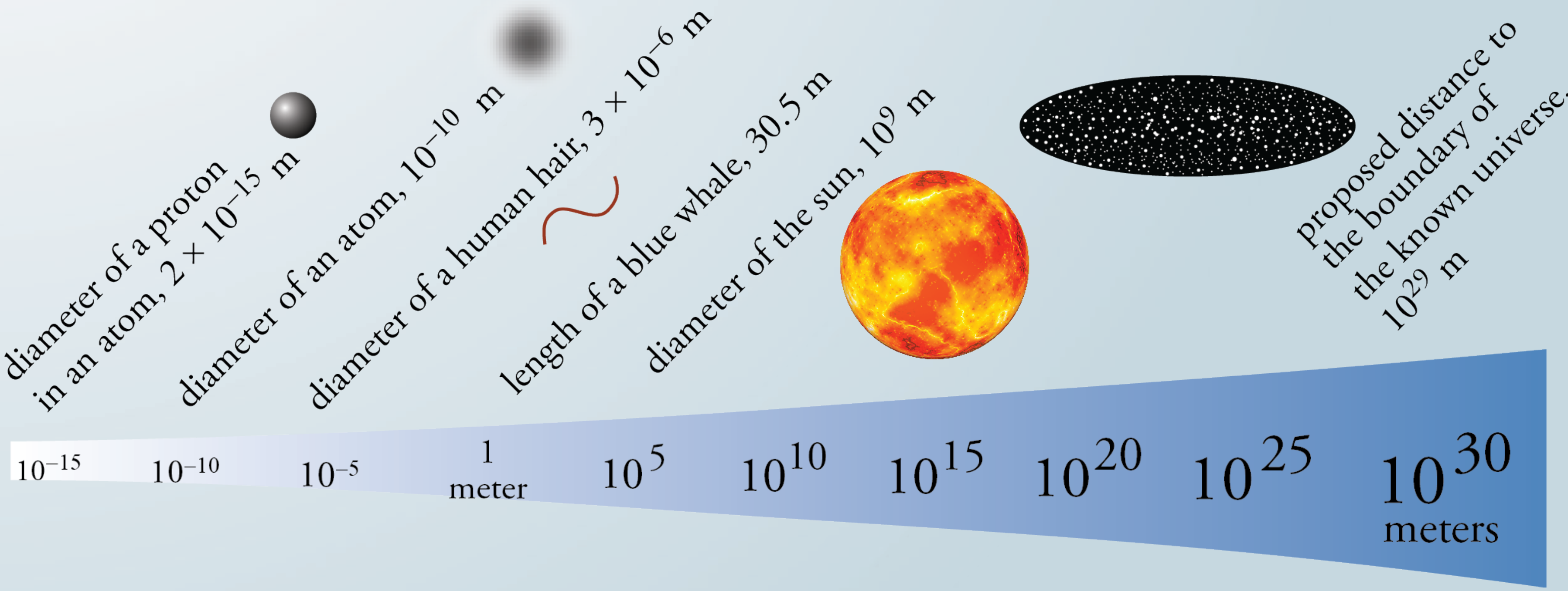
A mile is four times around a typical high school track.







Range of Lengths



Volume

1 fluid ounce (fl oz)

1 fl oz = 29.57 mL

1 mL = 0.03381 fl oz

1 milliliter (mL)
= about 20 drops

1 gal = 3.785 L

1 gallon (gal)
or 4 quarts (qt)

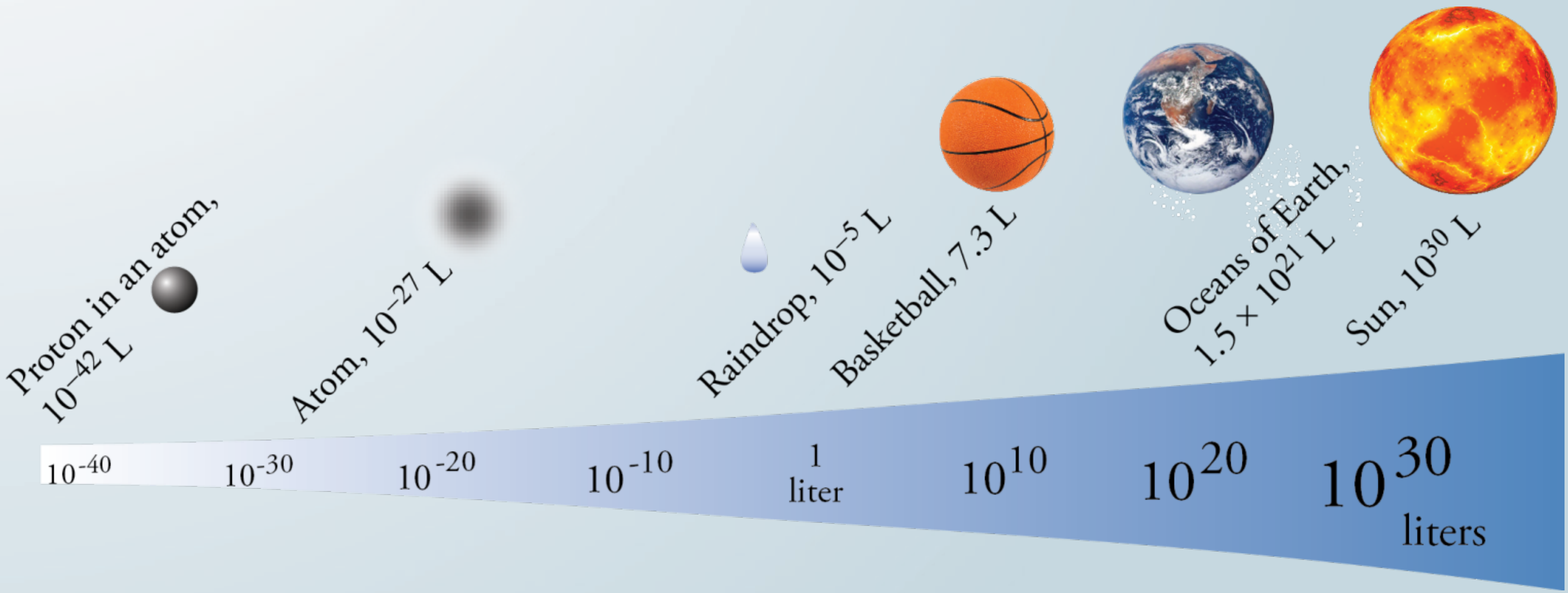
1 qt = 0.9464 L

1 qt or 32 fl oz

1 L = 1.057 qt
= 0.2642 gal

1 liter (L)
or 1000 mL

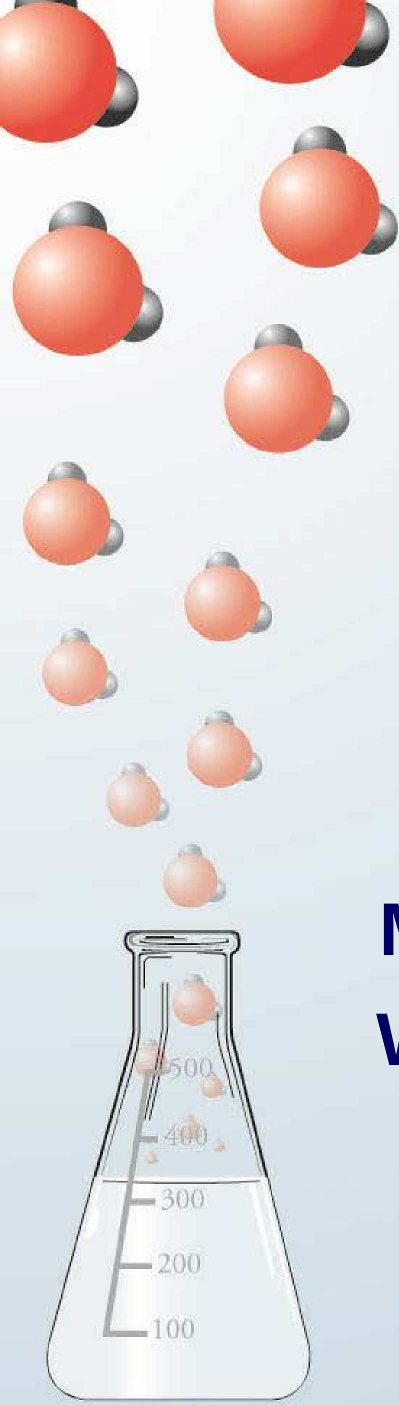
Range of Volumes



Mass and Weight

- ***Mass*** is usually defined as a measure of the amount of matter in an object. Mass can be defined as the property of matter that leads to gravitational attractions between objects and therefore gives rise to weight.
- ***Matter*** is anything that occupies a volume and has a mass.
- The ***weight*** of an object, on the Earth, is a measure of the force of gravitational attraction between the object and the Earth.

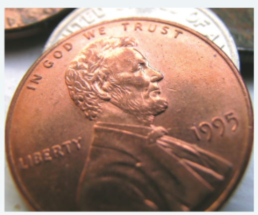
Comparison of the Mass and Weight of a 65 kg Person



	On Earth	Between Earth and Moon	On Moon
Mass	65 kg	65 kg	65 kg
Weight	637 N	≈0 N	1/6(637 N) = 106 N

Mass

1 oz = 28.35 g



About 2.5 grams (g) or about 0.088 ounce (oz)

1 lb = 453.6 g

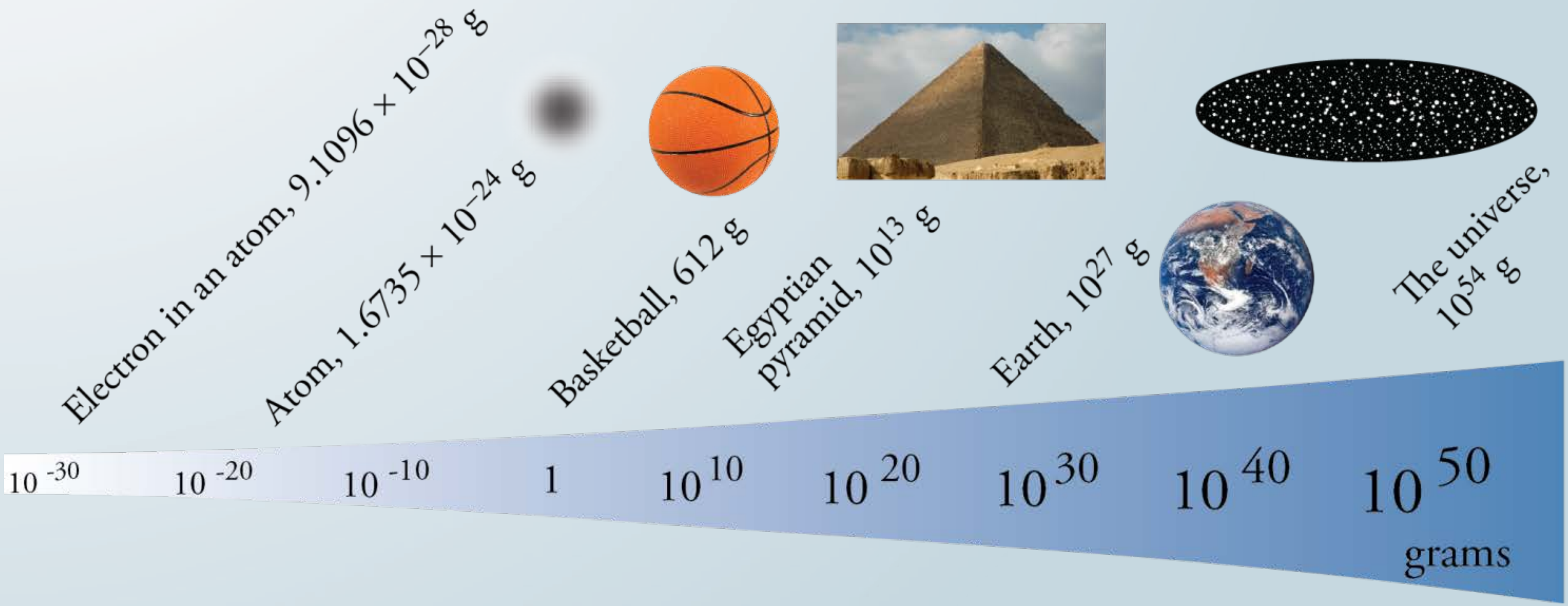
1 kg = 2.205 lb

About 1 kilogram (kg) or about 2.2 pounds (lb)

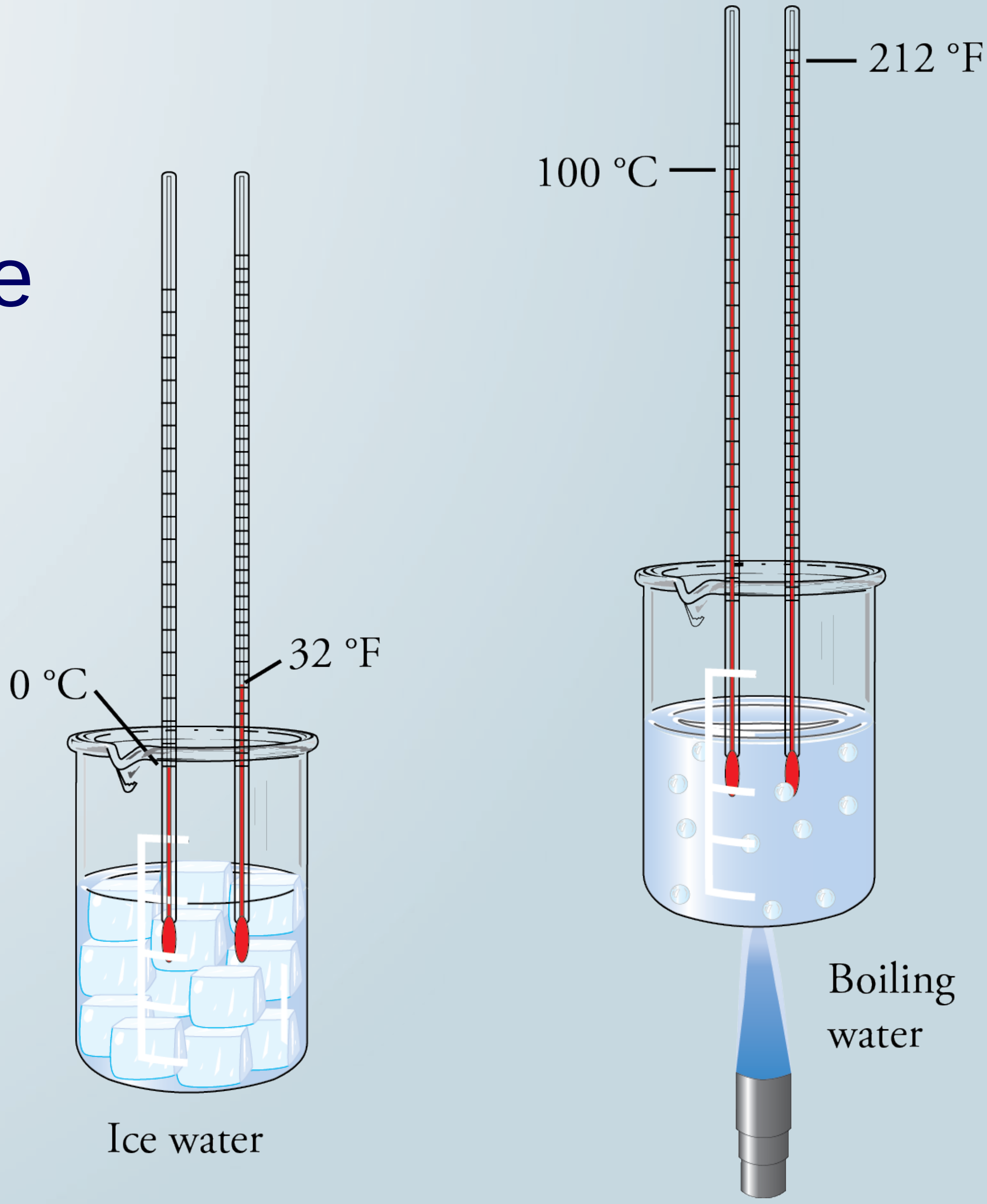
1 Mg = 1000 kg = 1 t

About 1 megagram (Mg) or 1 metric ton (t)

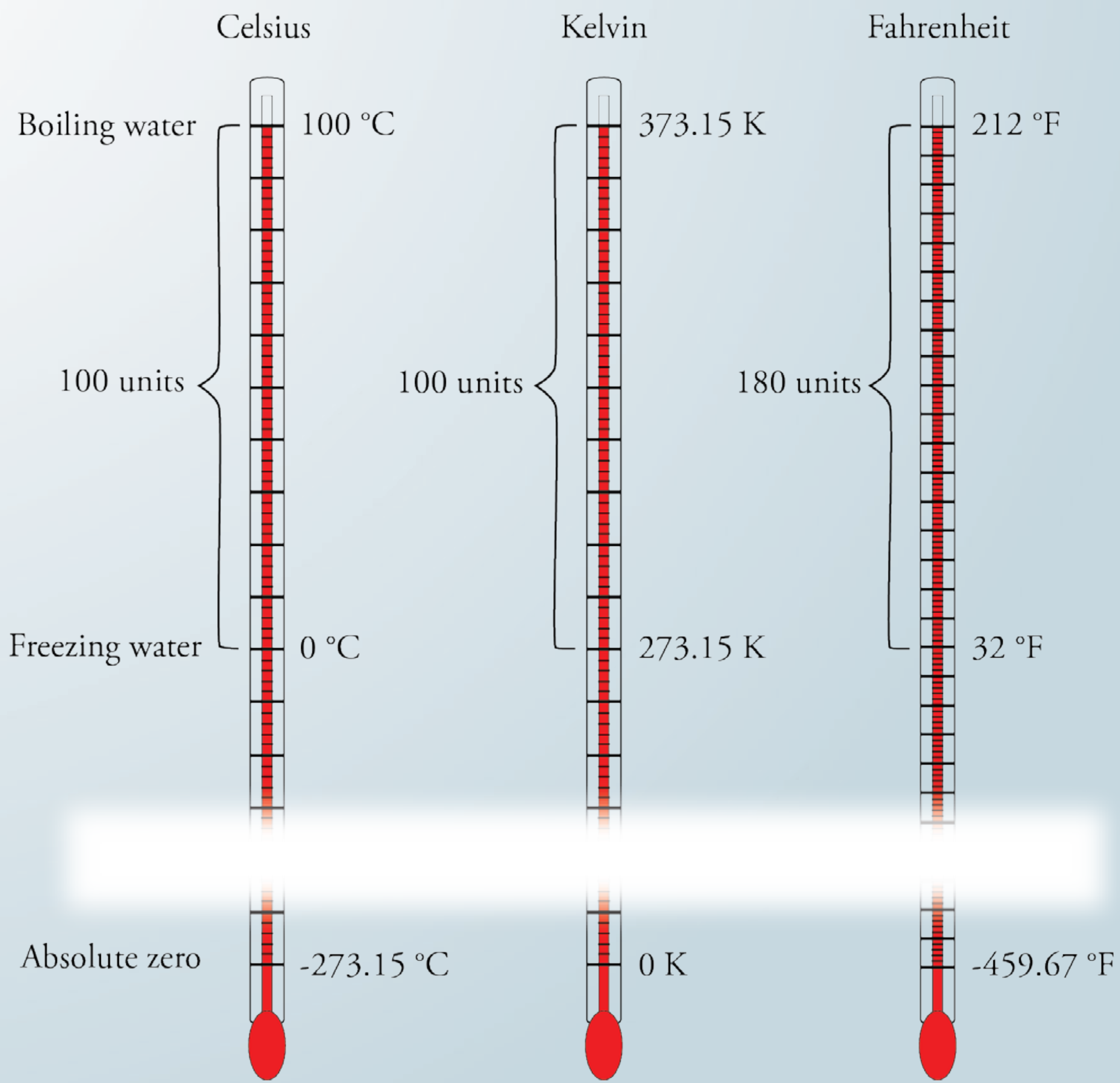
Range of Masses



Celsius and Fahrenheit Temperature



Comparing Temperature Scales



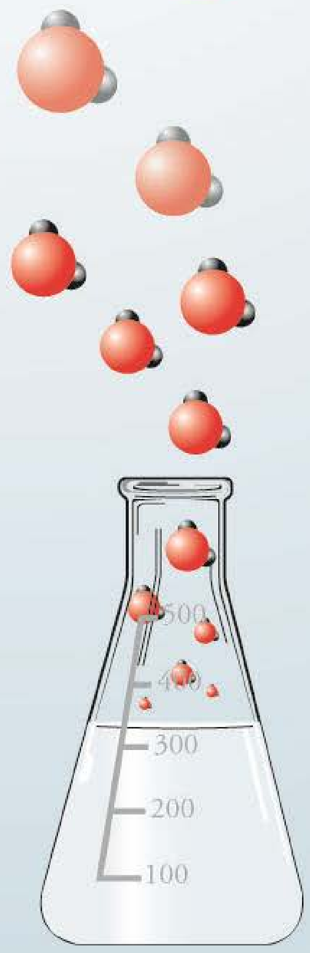
Temperature Conversions

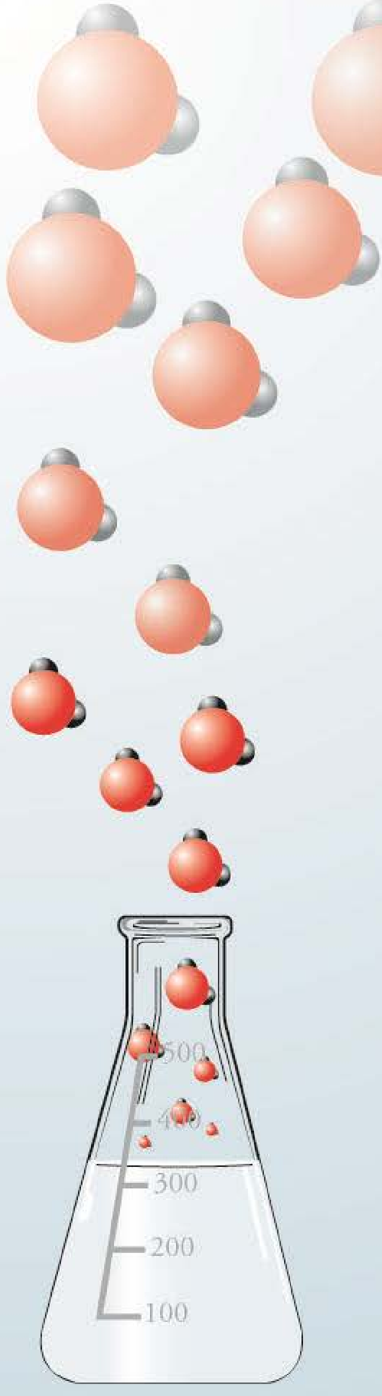
$$? \,^\circ F = \text{---} \,^\circ C \left(\frac{1.8 \,^\circ F}{1 \,^\circ C} \right) + 32 \,^\circ F$$

$$? \,^\circ C = \left(\text{---} \,^\circ F - 32 \,^\circ F \right) \left(\frac{1 \,^\circ C}{1.8 \,^\circ F} \right)$$

$$? \, K = \text{---} \,^\circ C + 273.15$$

$$? \,^\circ C = \text{---} \, K - 273.15$$





Calculations – Two Types

- **Unit conversions** (using unit analysis)
- **Equation-based calculations** (using algebra)

Our calculations are often a blend of the two.

Unit Conversions

All science requires mathematics. The knowledge of mathematical things is almost innate in us. . . [Mathematics] is the easiest of sciences, a fact which is obvious in that no one's brain rejects it…

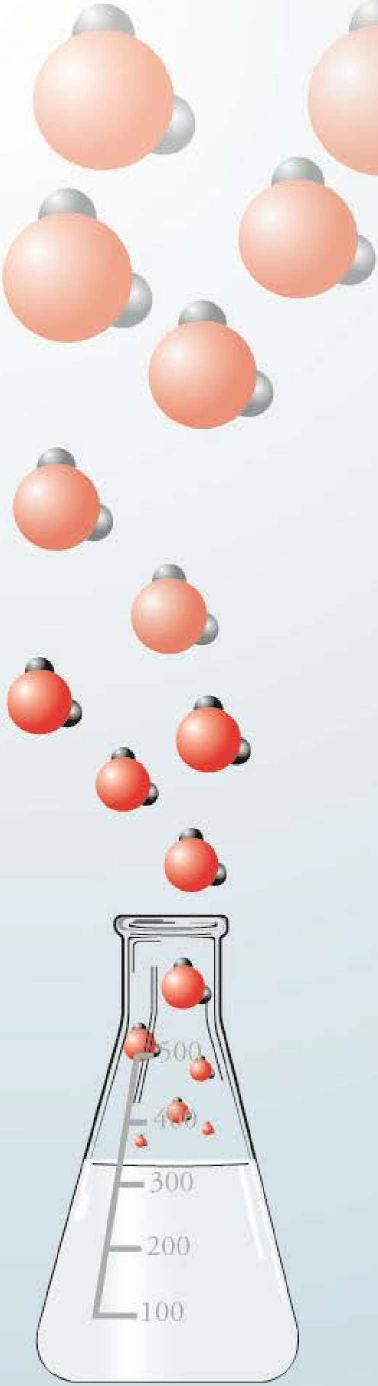
Roger Bacon (c. 1214-c. 1294)

Stand firm in your refusal to remain conscious during algebra. In real life, I assure you, there is no such thing as algebra.

Fran Lebowitz (b. 1951)

Problem: Using the following data, calculate the density of seawater in kg/L.

- The total mass of the hydrosphere is about 1.4×10^{15} Gg, which is about 0.023 percent of the Earth's total mass.
- Less than 3 percent is freshwater; the rest is saltwater, mostly in the ocean.
- The area of the world's oceans is 3.61×10^{8} km^2 and the volume is approximately 1.3×10^{9} km^3.



Unit Analysis Step 1

- **Step 1:** State your question in an expression that sets the unknown unit equal to the value given.
  - Start with the same number of units as you want.
    - If you want a single unit, start with a value that has a single unit.
    - If you want a ratio of two units, start with a value that has a ratio of two units, or start with a ratio of two values, each of which have one unit.
  - Put the correct type of unit in the correct position.

Unit Analysis Step 2

- **Step 2:** Multiply the expression to the right of the equals sign by one or more conversion factors that cancel the unwanted units and generate the desired unit.
  - If you are not certain which conversion factor to use, ask yourself, "What is the fundamental conversion and what conversion factor do I use for that type of conversion?"

Unit Analysis Steps 3 & 4

- **Step 3:** Check to be sure you used correct conversion factors and that your units cancel to yield the desired unit.
- **Step 4:** Do the calculation, rounding your answer to the correct number of significant figures and combining it with the correct unit.

Problem: Using the following data, calculate the density of seawater in kg/L.

- The total mass of the hydrosphere is about 1.4×10^{15} Gg, which is about 0.023 percent of the Earth's total mass.
- Less than 3 percent is freshwater; the rest is saltwater, mostly in the ocean.
- The area of the World Ocean is 3.61×10^{8} km^2 and its volume is approximately 1.3×10^{9} km^3.

$$\frac{?\ \text{kg ocean}}{\text{L ocean}} = \frac{1.4\times10^{15}\ \cancel{\text{Gg all water}}}{1.3\times10^{9}\ \cancel{\text{km}^3}\ \text{ocean}}\left(\frac{97\ \cancel{\text{Gg}}\ \text{ocean}}{100\ \cancel{\text{Gg all water}}}\right)\left(\frac{10^9\ \cancel{\text{g}}}{1\ \cancel{\text{Gg}}}\right)\left(\frac{1\ \text{kg}}{10^3\ \cancel{\text{g}}}\right)\left(\frac{1\ \cancel{\text{km}}}{10^3\ \cancel{\text{m}}}\right)^3\left(\frac{1\ \cancel{\text{m}^3}}{10^3\ \text{L}}\right)$$

$$= 1.04461538461538461538461538461 54\ \text{kg/L} \approx \mathbf{1.0\ kg/L}$$

English-Metric Conversion Factors

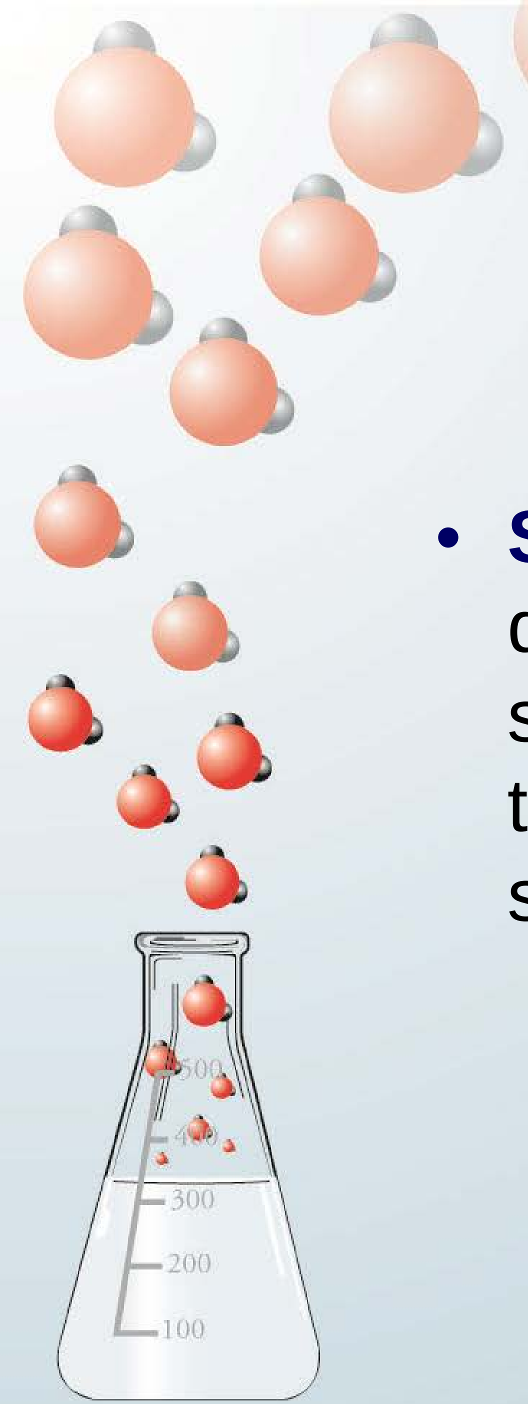
Type of Measurement	Probably Most Useful to Know	Others Useful to Know		
Length	$\frac{2.54\ cm}{1\ in.}$	$\frac{1.609\ km}{1\ mi}$	$\frac{39.37\ in.}{1\ m}$	$\frac{1.094\ yd}{1\ m}$
Mass	$\frac{453.6\ g}{1\ lb}$	$\frac{2.205\ lb}{1\ kg}$		
Volume	$\frac{3.785\ L}{1\ gal}$	$\frac{1.057\ qt}{1\ L}$		

Rounding Answers from Multiplication and Division Step 1

- **Step 1:** Determine whether each value is exact, and ignore exact values.
  - Exact values
    - Numbers that come from definitions are exact.
    - Numbers derived from counting are exact.
  - Do Step 2 for values that are not exact.
    - Values that come from measurements are never exact.
    - We will assume that values derived from calculations are not exact unless otherwise indicated.

Rounding Answers from Multiplication and Division Step 2

- **Step 2:** Determine the number of significant figures in each value that is not exact.
    - All non-zero digits are significant.
    - Zeros between nonzero digits are significant.
    - Zeros to the left of nonzero digits are not significant.
    - Zeros to the right of nonzero digits in numbers that include decimal points are significant.
    - Zeros to the right of nonzero digits in numbers without decimal points are ambiguous for significant figures.



Rounding Answers from Multiplication and Division Step 3

- **Step 3:** When multiplying and dividing, round your answer off to the same number of significant figures as the value used with the fewest significant figures.
  - If the digit to the right of the final digit you want to retain is less than 5, round down (the last digit remains the same).
  - If the digit to the right of the final digit you want to retain is 5 or greater, round up (the last significant digit increases by 1).

Rounding Answers from Addition and Subtraction

- **Step 1:** Determine whether each value is exact, and ignore exact values.
  - Skip exact values.
  - Do Step 2 for values that are not exact.
- **Step 2:** Determine the number of decimal positions for each value that is not exact.
- **Step 3:** Round your answer to the same number of decimal positions as the inexact value with the fewest decimal places.

Problem: Using the following data, calculate the approximate productivity of Earth's cultivated land in metric tons per hectare (Ha) per year.

- 15×10^6 km^2 cultivated land
- Global population is about 6.6×10^9 people
- Average food intake is 1 kg food per person per day
- 1 Ha = 10^4 m^2

Solution: Calculate the approximate productivity of Earth's cultivated land in metric tons per hectare (Ha) per year.

$$\frac{?\ \text{t}}{\text{Ha}\bullet\text{yr}} = \frac{6.6 \times 10^{9}\ \text{pers}}{1.5 \times 10^{7}\ \text{km}^2}\left(\frac{1\ \text{kg}}{\text{pers}\bullet\text{day}}\right)\left(\frac{1\ \text{t}}{10^{3}\ \text{kg}}\right)\left(\frac{1\ \text{km}}{10^{3}\ \text{m}}\right)^{2}\left(\frac{10^{4}\ \text{m}^2}{1\ \text{Ha}}\right)\left(\frac{365\ \text{day}}{1\ \text{yr}}\right)$$

$$= \frac{\mathbf{1.6\ t}}{\mathbf{Ha\bullet yr}}$$

Density

- ***Mass density*** is mass divided by volume. It is usually just called density. The most common units are g/mL or g/cm^3.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

- It can be used as a unit analysis conversion factor that converts mass to volume or volume to mass.
- **Specific gravity** = the ratio of the density of a substance to the density of water at 4 °C (1.000 g/cm^3)
  - Specific gravity is unitless.
- Other useful units for the density of water are

  = 1 kg/liter = 1 t/m^3 = 10^9 t/km^3

Percentage and Percentage Calculations

- Mass percentages and volume percentage can be used as unit analysis conversion factors to convert between units of the part and units of the whole.

For X% by mass $\dfrac{\text{X (any mass unit) part}}{\text{100 (same mass unit) whole}}$

For X% by volume $\dfrac{\text{X (any volume unit) part}}{\text{100 (same volume unit) whole}}$

Problem: Using the following data, calculate the approximate volume of the ocean in km^3 and mass in metric tons.

- Surface area of Earth = 5.1×10^8 km^2
- 70% of Earth covered by oceans
- Average depth of ocean about 4 km
- Density of water 1.0 g/cm^3

Solution: Calculate the approximate volume of the ocean in km^3 and mass in metric tons.

$V = A \cdot h$

$$? \text{ km}^3 \text{ ocean} = 5.1 \times 10^8 \, \cancel{\text{km}^2 \text{ total}} \left(\frac{70 \text{ km}^2 \text{ ocean}}{100 \, \cancel{\text{km}^2 \text{ total}}} \right) \cdot 4 \text{ km}$$

$$= \mathbf{1 \times 10^9 \text{ km}^3 \text{ ocean}}$$

$$? \text{ t ocean} = 1 \times 10^9 \, \cancel{\text{km}^3} \text{ ocean} \left(\frac{10^3 \, \cancel{\text{m}}}{1 \, \cancel{\text{km}}} \right)^3 \left(\frac{10^2 \, \cancel{\text{cm}}}{1 \, \cancel{\text{m}}} \right)^3 \left(\frac{1.0 \, \cancel{\text{g}}}{1 \, \cancel{\text{cm}^3}} \right) \left(\frac{1 \text{ t}}{10^6 \, \cancel{\text{g}}} \right)$$

$$= \mathbf{1 \times 10^{18} \text{ t ocean}}$$

Significance of Volume and Mass of Ocean

With all this volume and mass, why worry about polluting the ocean? - seems you could dilute anything to a very low concentration with all this water

- One reason is bioconcentration of pollutants in the food chain, which we'll discuss later in the course.
- Horizontally, most pollution goes into the shallow coastal zone where biodiversity and productivity are concentrated.
- Vertically, most pollution goes into the mixed layer.

Ocean Mixed Layer

- The ocean is not homogeneous, it is vertically stratified.
- The layer on top, which interacts most vigorously with the rest of the environment, is called the mixed layer.
- Mixed layer average depth = 80 m
- The mixed layer is the part of the ocean that…
  - is stirred by wave action
  - heated & illuminated by the sun;
  - exchanges heat, water, gases, & pollutants with the atmosphere; and
  - receives the inputs of the rivers.

“Something per Something”

- Anything that can be described as “something per something” can be written as a conversion factor and used to make conversions.

COW Ex 1 page 4: Calculate the number of cobblers in the U.S. using the following estimates.

- $20 per cobbler’s job
- Cobbler’s income of about $100,000 per year
- There’s about one cobbler job per person every four years
- There are about 310 million people in the U.S.

“Something per Something”

COW Ex 1 page 4: Calculate the number of cobblers in the U.S. using the following estimates.

- \$20 per cobbler’s bill
- Cobbler’s income of about \$100,000 per year
- There’s about one cobbler job per person every four years
- There are about 310 million people in the U.S.

$$? \text{ cobblers} = 3.1 \times 10^8 \cancel{\text{persons}} \left(\frac{1 \cancel{\text{job}}}{4 \cancel{\text{yr}} \bullet \cancel{\text{person}}} \right) \left(\frac{20 \cancel{\$}}{1 \cancel{\text{job}}} \right) \left(\frac{1 \cancel{\text{yr}} \bullet \text{cobbler}}{10^5 \cancel{\$}} \right)$$

$$= 15500 \text{ cobblers} \approx \mathbf{2 \times 10^4 \text{ cobblers}}$$

Equations for Early Calculations

Surface area of sphere $= 4\pi r^2 = 4\pi(d/2)^2 = \pi d^2$

$$\text{volume of a sphere} = \frac{4}{3}\pi r^3$$

circumference $= \pi \bullet$ diameter or $c = \pi d$

volume of a regular solid = area of the base × height

$V = A \times h$

$$V_{cyl} = A_{base} \times h = \pi r^2 \times h = \pi\left(\frac{d}{2}\right)^2 \times h = \frac{\pi d^2 h}{4}$$

$$\text{time} = \frac{\text{quantity of resource}}{\text{rate of consumption}}$$

$$\text{flow in} = \text{flow out} = \frac{\text{stock}}{\text{residence time}} = F_{in} = F_{out} = \frac{M}{T}$$

Mixed Problems: Equation and Unit Analysis

COW Ex 3 page 4: How many pairs of shoes can be made from a cow?

- Assume that both the cow and the shoe are spherical.
- Find the equation for the surface area of a sphere.

$$A = \pi d^2$$

- Estimate the diameter of a spherical cow and a spherical shoe…about 2 m and 0.15 m (15 cm or about 6 in.)
- Calculate the surface areas of a spherical cow and a spherical shoe, and express these as ratios.
- Use unit analysis to calculate the number of shoes per cow.

Mixed Problems: Equation and Unit Analysis

Solution - COW Ex 3 page 4: How many pairs of shoes can be made from a cow?

Surface area of sphere $= 4\pi r^2 = 4\pi(d/2)^2 = \pi d^2$

$d_{cow} \approx 2$ m

Surface area of spherical cow $= \pi d^2 = \pi(2\text{ m})^2 \approx \left(\dfrac{13\text{ m}^2}{1\text{ cow}}\right)$

$d_{shoe} \approx 0.15$ m (15 cm or about 6 in.)

Surface area of spherical shoe $= \pi d^2 = \pi(0.15\text{ m})^2 \approx \left(\dfrac{0.07\text{ m}^2}{1\text{ shoe}}\right)$

$$\frac{?\text{ shoe}}{\text{cow}} = \left(\frac{13\text{ m}^2}{1\text{ cow}}\right)\left(\frac{1\text{ shoe}}{0.07\text{ m}^2}\right) = 185.71\ldots\text{ shoes/cow} \approx \mathbf{2 \times 10^2\ shoes\ per\ cow}$$

Mixed Problems: Equation and Unit Analysis

COW Ex 1 page 8: If an asteroid created a crater 200 km in diameter, what would be its average depth?

- Earth area about 5.1×10^{14} m^2
- The dust from the asteroid led to about 0.02 g/cm^2 of all Earth's surface.
- About 20% of the asteroid's mass went to dust.
- 60 times as much earthly material blasted from crater zone as was contained in the asteroid.
- Density of earthly material is about 2 g/cm^3.

Mixed Problems: Equation and Unit Analysis

Solution - COW Ex 1 page 8: If an asteroid created a crater 200 km in diameter, what would be its average depth?

$$V_{cyl} = A_{base} \times h = \pi r^2 \times h = \pi \left(\frac{d}{2}\right)^2 \times h = \frac{\pi d^2 h}{4}$$

$$?\ \text{cm}^3\ \text{crater} =$$

$$5.1 \times 10^{14}\ \cancel{\text{m}^2}\ \text{earth} \left(\frac{10^2\ \cancel{\text{cm}}}{1\ \cancel{\text{m}}}\right)^{\cancel{2}} \left(\frac{0.02\ \cancel{\text{g ast. dust}}}{1\ \cancel{\text{cm}^2\ \text{earth}}}\right) \left(\frac{100\ \cancel{\text{g ast.}}}{20\ \cancel{\text{g ast. dust}}}\right) \left(\frac{60\ \cancel{\text{g earth}}}{1\ \cancel{\text{g ast.}}}\right) \left(\frac{1\ \text{cm}^3\ \text{earth}}{2\ \cancel{\text{g earth}}}\right)$$

$$= 1.5 \times 10^{19}\ \text{cm}^3\ \text{earth}$$

$$h = \frac{4V}{\pi d^2} = \frac{4(1.5 \times 10^{19}\ \cancel{\text{cm}^3})}{\pi(200\ \cancel{\text{km}})^2} \left(\frac{1\ \cancel{\text{m}}}{10^2\ \cancel{\text{cm}}}\right)^{\cancel{3}} \left(\frac{1\ \text{km}}{10^3\ \cancel{\text{m}}}\right)^{\cancel{3}}$$

$$= 0.48\ \text{km} \approx \mathbf{500\ m}$$